

# The Most Representative Composite Rank Ordering of Multi-Attribute Objects by the Particle Swarm Optimization

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**I. Introduction:** Consider  $n$  objects each with  $m < n$  common attributes. Suppose that these attributes cannot be measured, yet the objects can be rank ordered according to each attribute. More often than not, different evaluators would rank order the objects differently on the basis of each attribute or criterion (or even a particular evaluator may rank order the objects differently in different sessions of evaluation). There may be a good deal of concordance among the ranking scores obtained by the objects on the different criteria and the different sessions, but, in general, the concordance would not be perfect. There will be a need to summarize the ranking scores obtained on varied individual attributes (criteria). The summary will be given by a single array of overall ordinal ranking scores, which would represent the detailed attribute-(criterion-) wise ordinal ranking scores.

**II. Criterion of Representation:** Among the many possible criteria to summarize the imperfectly concordant arrays of individual measures  $(x_{ij} \in X_{(n,m)}; i = 1, 2, \dots, n; j = 1, 2, \dots, m)$  into a single array  $(z_i \in Z_{(n)}; i = 1, 2, \dots, n)$ , the one is to obtain  $Z$  such that the sum of squared (product moment) coefficients of correlation between the composite array of ranking scores,  $Z$ , with the individual arrays of ranking scores,  $x_j \in X$ , is maximum. Or, in other words,  $\sum_{j=1}^m r^2(Z, x_j)$  is maximum. It may be noted that this criterion also minimizes the (Euclidean) distance between  $Z$  and  $X$  such that  $Z$  passes through the center of the swarm of points in  $X$ . The product moment coefficient of correlation incorporates Spearman's coefficient of rank correlation as a special case.

**III. The Conventional Principal Component Analytic Approach:** However, as a matter of practice,  $Z$  is seldom found out so as to maximize  $\sum_{j=1}^m r^2(Z, x_j)$ . Instead,  $Y = Xw$  that maximizes  $\sum_{j=1}^m r^2(Y, x_j)$  is found out and, consequently,  $Y$  is rank ordered to obtain  $Z = \mathfrak{R}(Y)$ , where  $\mathfrak{R}(Y)$  is the rule of rank ordering  $Y$ . In order to do this, the Principal Components Analysis (Hotelling, 1933; Kendall and Stuart, 1968) is used which essentially runs into five steps: (i) standardization of  $x_j$  to  $u_j = (x_j - \bar{x}_j) / s_j \forall j = 1, 2, \dots, m$  where  $\bar{x}_j$  and  $s_j$  are the arithmetic mean and the standard deviation of  $x_j$  respectively; (ii) obtaining  $R = (1/n)U'U : u_j \in U$ ; (iii) obtaining the largest eigenvalue,  $\lambda$ , and the associated eigenvector,  $\omega$ , of  $R$ ; (iv) normalizing  $\omega$  such that  $v_i = \omega_i / \kappa$ , where  $\kappa = \left( \sum_{j=1}^m \omega_j^2 \right)^{1/2}$  and finally, (v) obtaining  $\Upsilon = Uv$ . Now, since the rank ordering of  $Y = Xw$  that maximizes  $\sum_{j=1}^m r^2(Y, x_j)$  is identical to the rank ordering obtained by  $\Upsilon = Uv$ , that is,  $\mathfrak{R}(Y)$  and  $\mathfrak{R}(\Upsilon)$  are identical, it is generally believed that rank ordering of objects on the basis of  $Y = Xw$  or  $\Upsilon = Uv$  best represents  $X$ . It may be shown, nevertheless, that  $Z = \mathfrak{R}(Y)$  or

$Z = \mathfrak{R}(Y)$  does not necessarily maximize  $\sum_{j=1}^m r^2(Z, x_j)$  and, thus, rank ordering based on the principal component analysis as described above is often sub-optimal (Mishra, 2008-b). This is obvious in view of the fact the  $Z = \mathfrak{R}(Y)$  or  $Z = \mathfrak{R}(Y)$  is not a linear function and consequently,  $Z$  may not inherit or preserve the optimality of  $Y$  (or  $Y$ ).

**IV. The Ordinal Principal Component Approach:** Korhonen (1984) and Korhonen and Siljamaki (1998) were perhaps the first attempts to directly obtain the overall ordinal ranking scores vector,  $Z$ , that maximizes  $\sum_{j=1}^m r^2(Z, x_j)$ . The authors named their method as the ‘ordinal principal component analysis’. In so doing, they used the constrained integer programming as a method of optimization (Li and Li, 2004). It is obvious that their approach to obtain the solution (rank ordering) may fail or become inordinately arduous if the scheme of rank ordering (Wikipedia, 2008-a) is standard competition ranking (1-2-2-4 rule), modified competition ranking (1-3-3-4 rule), dense ranking (1-2-2-3 rule) or fractional ranking (1-2.5-2.5-4 rule). This is so because the formulation of constraints in the integer programming problem with any ranking scheme other than the ordinal ranking (1-2-3-4 rule) would be extremely difficult or impracticable.

**V. Objectives of the Present Work:** In this paper we propose a new method to obtain  $Z$  that maximizes  $\|r(Z, x)\|_L$  irrespective of the choice of rank ordering scheme; it may be standard competition, modified competition, dense, fractional or ordinal. The matrix  $X$  may incorporate ordinal or cardinally measured variables. The norm,  $\|r(Z, x)\|_L$  could be absolute ( $L=1$ , maximizing  $\sum_{j=1}^m |r(Z, x_j)|$ ), Euclidean ( $L=2$ , maximizing  $[\sum_{j=1}^m r^2(Z, x_j)]^{1/2}$  or, by implication its square,  $\sum_{j=1}^m r^2(Z, x_j)$ ) or maximin ( $L=\infty$ ,  $\max(\min_j(|r(Z, x_j)|))$ ) or any other Minkowsky’s norm. The coefficient of correlation may be computed by Karl Pearson’s formula (of which the Spearman’s formula is only a special case) or Bradley’s formula of absolute correlation (Bradley, 1985). Different measures of norm as well as correlation may have different implications as well as applications.

**VI. The Method of Optimization:** A choice of method of optimization depends much on the objective function (and the constraints, if any). We have proposed a very general objective function that may be smooth, kinky or even abruptly changing, depending on the norm chosen. Further, nonlinear functions such as computation of correlation coefficient and rank ordering are imbedded in the objective function. In view of these complications, we have chosen the (Repulsive) Particle Swarm method of optimization.

**VI(i). The Particle Swarm Optimizer:** The Particle Swarm method of optimization (Eberhart and Kennedy, 1995) is a population-based, stochastic search method that does not require derivatives of the optimand function to be computed. This method is also an instance of a successful application of the philosophy of decentralized decision-making and *bounded rationality* to solve the global optimization problems (Hayek, 1948, 1952; Simon, 1982; Bauer, 2002; Fleischer, 2005). It is observed that a swarm of birds or insects or a school of fish searches for food, protection, etc. in a very typical manner. If one of the members of the swarm sees a desirable path to go, the others in the swarm will follow it. Every member of the swarm searches for the best in its locality - learns from its own experience. Additionally, each member

learns from the others, typically from the best performer among them. Even human beings show a tendency to learn from their own experience, their immediate neighbours and the ideal performers.

The Particle Swarm method of optimization mimics the said behaviour (Wikipedia, 2008-c). Every individual of the swarm is considered as a particle in a multidimensional space that has a position and a velocity. These particles fly through hyperspace and remember the best position that they have seen. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions. There are two main ways this communication is done: (i) "swarm best" that is known to all (ii) "local bests" are known in neighborhoods of particles. Updating the position and velocity is done at each iteration as follows:

$$v_{i+1} = \omega v_i + c_1 r_1 (\hat{x}_i - x_i) + c_2 r_2 (\hat{x}_{gi} - x_i)$$

$$x_{i+1} = x_i + v_{i+1}$$

where,

- $x$  is the position and  $v$  is the velocity of the individual particle. The subscripts  $i$  and  $i+1$  stand for the recent and the next (future) iterations, respectively.
- $\omega$  is the inertial constant. Good values are usually slightly less than 1.
- $c_1$  and  $c_2$  are constants that say how much the particle is directed towards good positions. Good values are usually right around 1.
- $r_1$  and  $r_2$  are random values in the range  $[0,1]$ .
- $\hat{x}$  is the best that the particle has seen.
- $\hat{x}_g$  is the global best seen by the swarm. This can be replaced by  $\hat{x}_L$ , the local best, if neighborhoods are being used.

The Particle Swarm method has many variants. The Repulsive Particle Swarm (RPS) method of optimization (Urfalioglu, 2004), the one of such variants, is particularly effective in finding out the global optimum in very complex search spaces (although it may be slower on certain types of optimization problems). Other variants use a dynamic scheme (Liang and Suganthan, 2005). In the traditional RPS the future velocity,  $v_{i+1}$  of a particle at position with a recent velocity,  $v_i$ , and the position of the particle are calculated by:

$$v_{i+1} = \omega v_i + \alpha r_1 (\hat{x}_i - x_i) + \omega \beta r_2 (\hat{x}_{hi} - x_i) + \omega \gamma r_3 z$$

$$x_{i+1} = x_i + v_{i+1}$$

where,

- $x$  is the position and  $v$  is the velocity of the individual particle. The subscripts  $i$  and  $i+1$  stand for the recent and the next (future) iterations, respectively.
- $r_1, r_2, r_3$  are random numbers,  $[0,1]$
- $\omega$  is inertia weight,  $[0.01,0.7]$
- $\hat{x}$  is the best position of a particle
- $x_h$  is best position of a randomly chosen other particle from within the swarm
- $z$  is a random velocity vector

- $\alpha$ ,  $\beta$ ,  $\gamma$  are constants

Occasionally, when the process is caught in a local optimum, some *chaotic* perturbation in position as well as velocity of some particle(s) may be needed.

**VI(ii). Memetic Modifications in the RPS Method:** The traditional RPS gives little scope of local search to the particles. They are guided by their past experience and the communication received from the others in the swarm. We have modified the traditional RPS method by endowing stronger (wider) local search ability to each particle. Each particle flies in its local surrounding and searches for a better solution. The domain of its search is controlled by a new parameter. This local search has no preference to gradients in any direction and resembles closely to tunneling. This added exploration capability of the particles brings the RPS method closer to what we observe in real life. However, in some cases moderately wide search works better. This local search capability endowed to the individual members of the swarm makes the RPS somewhat memetic (in the sense of Dawkins, 1976 and Ong et al., 2006).

It has been said that each particle learns from its ‘chosen’ inmates in the swarm. Now, at the one extreme is to learn from the best performer in the entire swarm. This is how the particles in the original PS method learn. However, such learning is not natural. How can we expect the individuals to know as to the best performer and interact with all others in the swarm? We believe in limited interaction and limited knowledge that any individual can possess and acquire. So, our particles do not know the ‘best’ in the swarm. Nevertheless, they interact with some chosen inmates that belong to the swarm. Now, the issue is: how does the particle choose its inmates? One of the possibilities is that it chooses the inmates closer (at lesser distance) to it. But, since our particle explores the locality by itself, it is likely that it would not benefit much from the inmates closer to it. Other relevant topologies are : (the celebrated) *ring topology*, ring topology hybridized with random topology, star topology, von Neumann topology, etc.

Let us visualize the possibilities of choosing (a predetermined number of) inmates randomly from among the members of the swarm. This is much closer to reality in the human world. When we are exposed to the mass media, we experience this. Alternatively, we may visualize our particles visiting a public place (e.g. railway platform, church, etc) where it (he) meets people coming from different places. Here, geographical distance of an individual from the others is not important. Important is how the experiences of others are communicated to us. There are large many sources of such information, each one being selective in what it broadcasts and each of us selective in what we attend to and, therefore, receive. This selectiveness at both ends transcends the geographical boundaries and each one of us is practically exposed to randomized information. Of course, two individuals may have a few common sources of information. We have used these arguments in the scheme of dissemination of others’ experiences to each individual particle. Presently, we have assumed that each particle chooses a pre-assigned number of inmates (randomly) from among the members of the swarm. However, this number may be randomized to lie between two pre-assigned limits.

**VII. A Formal Description of the Problem:** Now we formally describe our problem of rank ordering the individuals characterized by  $X$  as follows:

Maximize,

$$f_1 = |r(Z, x_1)| + |r(Z, x_2)| + \dots + |r(Z, x_m)| \text{ or}$$

$$f_2 = r^2(Z, x_1) + r^2(Z, x_2) + \dots + r^2(Z, x_m) \text{ or}$$

$$f_s = \max[\min |r(Z, x_1)|, |r(Z, x_2)|, \dots, |r(Z, x_m)|] ,$$

whichever the choice may be, such that  $Z$  is an array of ranking scores obtained by the individuals described by  $X$ , following a suitable scheme of rank ordering (such as the standard competition ranking, the dense ranking, or the ordinal ranking, etc) and the correlation function,  $r(Z, x_j)$ , is computed by a suitable formula (Karl Pearson's product moment or Bradley's absolute correlation). It is obvious that the optimand objective function  $f_1$ ,  $f_2$  or  $f_s$  is defined in terms of two procedures: (i)  $Z \leftarrow X$ , and (ii)  $r \leftarrow (Z, x_j)$ . In this sense, the optimand function is unusual and involves logico-arithmetic operations rather than simple arithmetic operations. This is unlike the formulation by Korhonen and Siljamaki (1998) who, by means of imposing constraints on the elements of  $Z$ , could convert the problem of optimization into a purely arithmetic procedure.

**VIII. A Computer Program:** We have developed a computer program (in FORTRAN) to solve the problem. It consists of a main program and 13 subroutines. The subroutines RPS, LSRCH, NEIGHBOR, FSELECT, RANDOM and FUNC are used for the purpose of optimization. The subroutine GINI computes the degree of diversity in the swarm population on reaching the optimal solution by some members of the swarm. Other subroutines relate to rank ordering (DORANK) and computing the coefficient of correlation. In particular, the subroutine CORA computes Bradley's absolute correlation (Bradley, 1985; Mishra, 2008-a). The parameters NOB and MVAR (no. of observations,  $n$ , and no. of variables,  $m$ , in  $X_{(n,m)}$ ) need to be specified in the main program as well as in the subroutine CORD. In the subroutine DORANK the scheme of rank ordering should be specified (whether rank ordering is to be done by 1-2-3-4 rule, 1-2-2-4 rule, 1-3-3-4 rule, 1-2-2-3 rule or 1-2.5-2.5-4 rule). Presently, it is set to NRL=0 for the ordinal (1-2-3-4 ranking) rule. Parameters in other programs usually do not need re-specification. However, necessary comments have been given to change them if so needed in very special conditions.

**IX. Three Examples of Sub-optimality of the PCA-based Rank ordering:** In order to illustrate the method of the most representative composite rank ordering suggested by us, the program developed for the same purpose, and the superiority of our method to the PCA-based rank ordering, we present three examples. All the three examples are simulated by us. Notation-wise, we use  $Y = Xw$  and  $Z_1 = \mathfrak{R}(Y)$  obtained by maximization of  $\sum_{j=1}^m r^2(Y, x_j)$  resulting into the PCA-based ranking scores,  $Z_1$ . Analogously, we use  $Y' = Xv$  and  $Z_2 = \mathfrak{R}(Y')$  obtained by maximization of  $\sum_{j=1}^m r^2(Z_2, x_j)$  resulting into the most optimal ranking scores,  $Z_2$ , proposed by us in this paper. These examples clearly demonstrate that the PCA-based  $Z_1$  is sub-optimal.

**IX(i). Example-1:** The simulated dataset ( $X$ ) on ranking scores of 30 candidates awarded by 7 evaluators, the results obtained by running the principal component algorithm (PCA) and the overall rankings based on the same ( $Y$  and  $Z_1$ ) and the results of rank order optimization exercise based on our method ( $Y'$  and  $Z_2$ ) are presented in Table-1.1. In table-1.2 are presented

the inter-correlation matrix,  $R_1$ , for the variables  $[Z_1, x_1, x_2, x_3, x_4, x_5, x_6, x_7]$ . The last two rows of Table-1.2 are the weight ( $w$ ) vector used to obtain  $Y = Xw$  and component loadings, that is,  $r(Y, x_j)$ . The sum of squared component loadings  $(S_1) = 4.352171$ . The measure of representativeness of  $Z_1$  that is  $F_1 = \sum_{j=1}^7 r^2(Z_1, x_j) = 4.287558$ . All these results pertain to the standard PCA, obtained by direct optimization. These results compare perfectly with those obtained by STATISTICA, a standard statistical software, that uses the conventional singular value decomposition method to obtain the PCA-based component scores.

In Table-1.3 we have presented the inter-correlation matrix,  $R_2$ , for variables  $[Z_2, x_1, x_2, x_3, x_4, x_5, x_6, x_7]$ , weights and the component loadings when the same dataset (as mentioned above) is subjected to the direct maximization of  $\sum_{j=1}^7 r^2(Z_2, x_j)$ . The weights and the component loadings relate to  $Y' = Xv$  and  $r(Z_2, x_j)$ . The sum of squared component loadings  $(S_2) = 4.287902$  and the measure of representativeness of  $Z_2$  that is  $F_2 = \sum_{j=1}^7 r^2(Z_2, x_j)$  also is 4.287902. Since  $F_2 > F_1$ , the sub-optimality of the PC-based  $F_1$  for this dataset is demonstrated. Notably, the candidates #8, #20, #21 and #26 are rank ordered differently by the two methods. It may be noted that the changed rank ordering may mean a lot to the candidates.

**IX(ii). Example-2:** The simulated data and  $Y, Y', Z_1$  and  $Z_2$  for this dataset are presented in Table-2.1. The inter-correlation matrices,  $R_1$  and  $R_2$  and the associated weights and factor loadings also are presented in Tables-2.2 and 2.3. The values of  $F_1$  and  $F_2$  for this dataset are 2.610741 and 2.610967 respectively. This also shows the sub-optimality of the PC-based  $F_1$ . The candidates #2, #5, #12, #13, #14 and #30 are rank ordered differently by the two methods.

**IX(iii). Example-3:** One more simulated dataset and  $Y, Y', Z_1$  and  $Z_2$  for this dataset are presented in Table-3.1. The inter-correlation matrices,  $R_1$  and  $R_2$  and the associated weights and factor loadings also are presented in Tables-3.2 and 3.3. The values of  $F_1$  and  $F_2$  for this dataset are 4.476465 and 4.476555 respectively. Once again, it is demonstrated that the PC-based  $F_1$  is sub-optimal. The candidates #22 and #26 are rank ordered differently by the two methods.

**X. Two Examples of Overall Rank ordering by Maximization of the Absolute Norm:** Earlier it has been mentioned that an overall composite rankings may also be obtained by maximization of  $f_1 = |r(Z_2, x_1)| + |r(Z_2, x_2)| + \dots + |r(Z_2, x_m)|$  which is only an analogous version of maximization of  $f_2 = r^2(Z_2, x_1) + r^2(Z_2, x_2) + \dots + r^2(Z_2, x_m)$ . Similarly, analogous to the principal component based rank ordering scores  $Z_1 = \mathfrak{R}(Y); Y = Xw$  obtained by maximization of  $\sum_{j=1}^m r^2(Y, x_j)$ , one may also obtain  $Z_1'' = \mathfrak{R}(Y''); Y'' = Xv$  by maximization of  $\sum_{j=1}^m |r(Y'', x_j)|$ . This exercise has been done here and two examples have been presented. Results of examples 4 and 5 are presented in the Tables 4.1 through 5.3. The solutions exhibit some robustness to large variations of scores obtained by different individuals. We also find that  $AF_1 (= \sum_{j=1}^m |r(Y'', x_j)|)$  yielding  $Z_1'' = \mathfrak{R}(Y''); Y'' = Xv$  in the Table 4.2 (and 5.2) and  $AF_2 (= \sum_{j=1}^m r^2(Z_2'', x_j))$  yielding  $Z_2''$ , in which  $Y''' = X\omega$  is instrumental to obtain  $Z_2'' = \mathfrak{R}(Y''')$  in the Table 4.3 (and 5.3) are equal, although  $Z_1''$  and  $Z_2''$  rank order the objects differently (see objects #11 and #12 in Table 4.1 and objects #9 and #16 in Table 5.1). This equality suggests that maximization of the absolute norm yields

multiple solutions. Absolute norm estimators often exhibit this property of multiple solutions (Wikipedia, 2008-b). In the sense of sum of squared component loadings ( $F_1$  and  $F_2$ ),  $Z_2''$  performs better than  $Z_1''$  in example 4, but worse in example 5, although this is a different matter altogether. Obviously, under such conditions, no clear conclusion can be drawn.

**XI. An Example of Overall Rank ordering by Maximin Absolute Correlation Criterion:** In Tables 6.1 through 6.3 we present the results of an exercise to obtaining the composite rank ordering on the basis of maximin (absolute) correlation. Such maximin correlation signifies the floor (lowest absolute) correlation that the individual ranking scores ( $X$ ) may have with the overall composite ranking score. In table 6.1,  $Z_1^*$  is obtained by  $\max(\min(|r(Y^*, x_j)|))$  while  $Z_2^*$  is obtained by  $\max(\min(|r(Z_2^*, x_j)|))$ . The maximin correlation for  $Z_1^*$  is 0.671190, smaller than the maximin correlation (0.673860) for  $Z_2^*$ . Once again, sub-optimality of  $Z_1^*$  is demonstrated. Representation of  $X$  by the composite ranking scores has been presented in Fig.-1. It may also be reported that in obtaining the overall rankings by maximin correlation, the optimization method (the RPS) is often caught in the local optimum trap and, hence, the program was run several times with different seeds for generating random numbers.

**XII. Concluding Remarks:** Rank-ordering of individuals or objects on multiple criteria has many important practical applications. A reasonably representative composite rank ordering of multi-attribute objects/individuals or multi-dimensional points is often obtained by the Principal Component Analysis, although much inferior but computationally convenient methods also are frequently used. However, such rank ordering – even the one based on the Principal Component Analysis – may not be optimal. This has been demonstrated by several numerical examples. To solve this problem, the Ordinal Principal Component Analysis was suggested some time back. However, this approach cannot deal with various types of alternative schemes of rank ordering, mainly due to its dependence on the method of solution by the constrained integer programming. In this paper we propose an alternative method of solution, namely by the Particle Swarm Optimization. A computer program in FORTRAN to solve the problem has also been provided. The suggested method is notably versatile and can take care of various schemes of rank ordering, norms and types or measures of correlation. The versatility of the method and its capability to obtain the most representative composite rank ordering of multi-attribute objects or multi-dimensional points have been demonstrated by several numerical examples. It has also been found that rank ordering based on maximization of the sum of absolute values of the correlation coefficients of composite rank scores with its constituent variables has robustness, but it may have multiple optimal solutions. Thus, while it solves the one problem, it gives rise to the other problem. On this consideration, rank ordering by optimization of the absolute norm cannot be readily prescribed. The overall ranking of objects by maximin correlation principle performs better if the composite rank scores are directly obtained by maximization of  $\min(|r(Z_2^*, x_j)|)$  rather than  $\min(|r(Y^*, x_j)|)$ .

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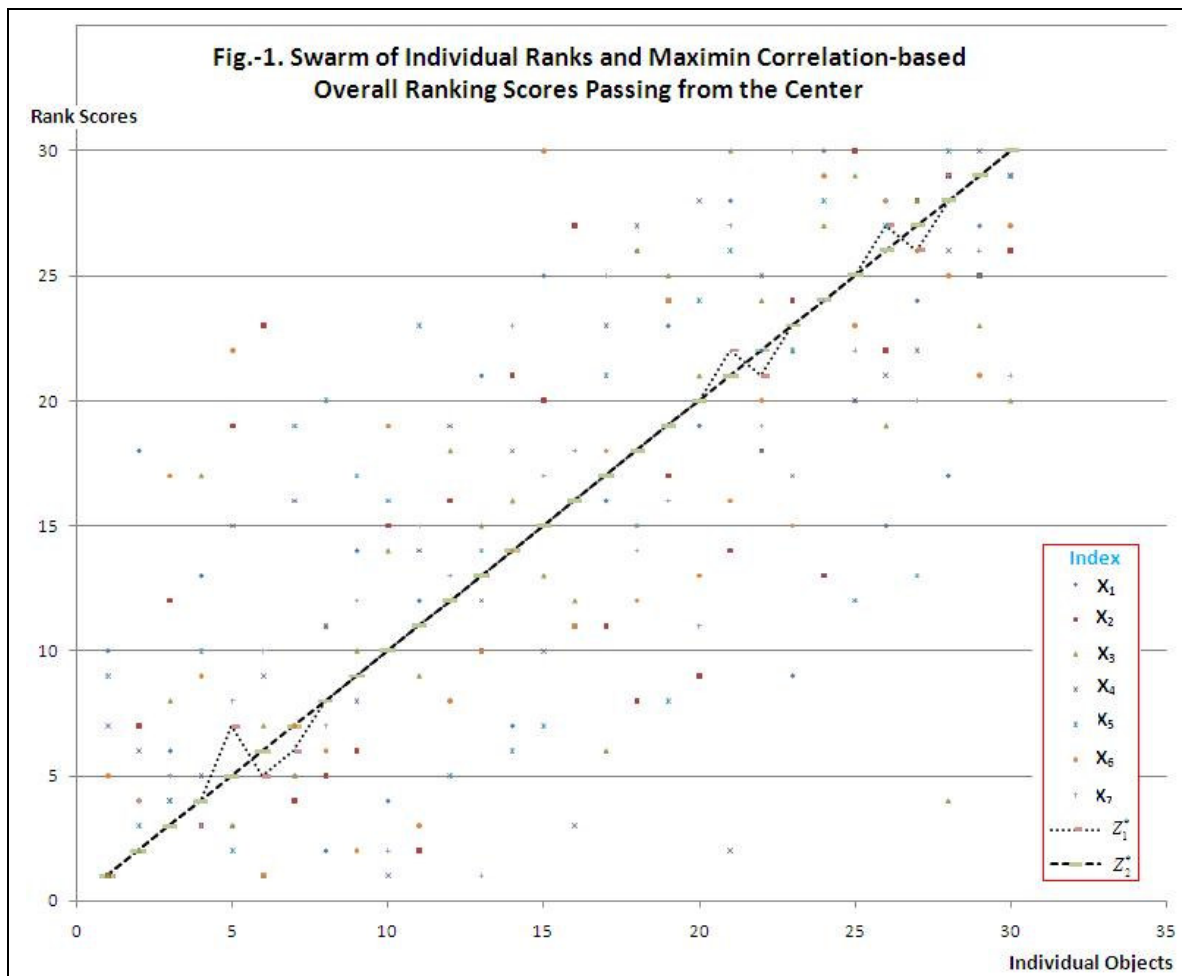
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| Sl. No. | Ranking Scores of 30 candidates awarded by Seven Evaluators |       |       |       |       |       |       | Composite Score (Y) Optimized Results |       | Rank-Order ( $Z_2$ ) Optimized Results |       |
|---------|---|-------|-------|-------|-------|-------|-------|---------------------------------------|-------|--|-------|
|         | $X_1$   | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | Y                                     | $Z_1$ | Y'                                     | $Z_2$ |
| 1       | 1   | 10    | 3     | 1     | 1     | 6     | 8     | 11.22449                              | 3     | 9.94513                                | 3     |
| 2       | 4   | 9     | 12    | 14    | 11    | 5     | 1     | 21.21789                              | 5     | 22.02743                               | 5     |
| 3       | 28  | 18    | 20    | 25    | 27    | 15    | 30    | 61.89772                              | 26    | 60.65756                               | 26    |
| 4       | 23  | 29    | 15    | 18    | 30    | 17    | 29    | 60.44523                              | 25    | 57.66703                               | 25    |
| 5       | 11  | 19    | 18    | 26    | 20    | 23    | 26    | 54.19009                              | 22    | 52.54342                               | 22    |
| 6       | 26  | 27    | 28    | 24    | 29    | 28    | 18    | 67.29577                              | 28    | 65.48724                               | 28    |
| 7       | 18  | 25    | 30    | 21    | 16    | 18    | 24    | 57.61014                              | 24    | 56.18663                               | 24    |
| 8       | 8   | 16    | 9     | 15    | 15    | 27    | 12    | 37.83847                              | 12    | 35.71465                               | 11    |
| 9       | 5   | 21    | 26    | 23    | 23    | 9     | 15    | 46.22725                              | 19    | 46.03365                               | 19    |
| 10      | 16  | 17    | 11    | 16    | 14    | 20    | 19    | 42.55687                              | 16    | 40.82902                               | 16    |
| 11      | 22  | 15    | 21    | 20    | 17    | 19    | 13    | 47.86438                              | 20    | 47.36776                               | 20    |
| 12      | 25  | 12    | 22    | 22    | 19    | 30    | 21    | 56.74538                              | 23    | 54.95989                               | 23    |
| 13      | 15  | 23    | 16    | 27    | 10    | 8     | 14    | 43.60463                              | 17    | 44.80203                               | 17    |
| 14      | 21  | 4     | 25    | 9     | 22    | 16    | 16    | 42.06405                              | 15    | 40.18355                               | 15    |
| 15      | 24  | 26    | 27    | 28    | 13    | 29    | 25    | 65.25869                              | 27    | 63.94859                               | 27    |
| 16      | 29  | 24    | 29    | 30    | 21    | 24    | 28    | 70.25945                              | 30    | 69.24504                               | 30    |
| 17      | 3   | 8     | 13    | 8     | 3     | 13    | 17    | 24.69311                              | 7     | 23.04507                               | 7     |
| 18      | 12  | 30    | 14    | 12    | 12    | 14    | 11    | 39.33113                              | 14    | 37.96445                               | 14    |
| 19      | 14  | 1     | 5     | 3     | 2     | 1     | 9     | 13.52052                              | 4     | 13.35966                               | 4     |
| 20      | 17  | 5     | 7     | 17    | 8     | 26    | 20    | 37.82411                              | 11    | 36.15267                               | 12    |
| 21      | 2   | 3     | 1     | 2     | 4     | 12    | 4     | 10.12458                              | 2     | 8.83153                                | 1     |
| 22      | 20  | 28    | 8     | 13    | 25    | 21    | 23    | 51.38818                              | 21    | 48.22206                               | 21    |
| 23      | 10  | 7     | 6     | 7     | 9     | 22    | 5     | 24.16536                              | 6     | 22.50172                               | 6     |
| 24      | 9   | 14    | 17    | 5     | 18    | 2     | 2     | 24.73648                              | 8     | 24.29566                               | 8     |
| 25      | 30  | 20    | 23    | 19    | 6     | 11    | 6     | 43.85996                              | 18    | 45.06863                               | 18    |
| 26      | 6   | 2     | 2     | 4     | 7     | 3     | 3     | 10.08922                              | 1     | 9.92610                                | 2     |
| 27      | 13  | 13    | 10    | 11    | 28    | 7     | 22    | 38.94103                              | 13    | 36.86430                               | 13    |
| 28      | 19  | 6     | 19    | 6     | 5     | 10    | 7     | 27.10137                              | 10    | 26.67865                               | 10    |
| 29      | 27  | 22    | 24    | 29    | 26    | 25    | 27    | 68.06338                              | 29    | 66.67034                               | 29    |
| 30      | 7   | 11    | 4     | 10    | 24    | 4     | 10    | 26.03013                              | 9     | 25.10108                               | 9     |

|          | $Z_1$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_1$    | 1.000000        | <b>0.805117</b> | <b>0.781980</b> | <b>0.801112</b> | <b>0.891880</b> | <b>0.690768</b> | <b>0.658287</b> | <b>0.824694</b> |
| $X_1$    | <b>0.805117</b> | 1.000000        | 0.474082        | 0.666741        | 0.645384        | 0.451390        | 0.537709        | 0.597330        |
| $X_2$    | <b>0.781980</b> | 0.474082        | 1.000000        | 0.554616        | 0.688543        | 0.552392        | 0.409121        | 0.569299        |
| $X_3$    | <b>0.801112</b> | 0.666741        | 0.554616        | 1.000000        | 0.731257        | 0.438487        | 0.426919        | 0.491880        |
| $X_4$    | <b>0.891880</b> | 0.645384        | 0.688543        | 0.731257        | 1.000000        | 0.526140        | 0.606229        | 0.708120        |
| $X_5$    | <b>0.690768</b> | 0.451390        | 0.552392        | 0.438487        | 0.526140        | 1.000000        | 0.324583        | 0.630256        |
| $X_6$    | <b>0.658287</b> | 0.537709        | 0.409121        | 0.426919        | 0.606229        | 0.324583        | 1.000000        | 0.608009        |
| $X_7$    | <b>0.824694</b> | 0.597330        | 0.569299        | 0.491880        | 0.708120        | 0.630256        | 0.608009        | 1.000000        |
| Weights  |                 | 0.381529        | 0.369988        | 0.377232        | 0.430731        | 0.337587        | 0.337887        | 0.401968        |
| Loadings |                 | 0.796004        | 0.771847        | 0.786981        | 0.898610        | 0.704269        | 0.704900        | 0.838500        |

|          | $Z_2$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_2$    | 1.000000        | <b>0.810901</b> | <b>0.776641</b> | <b>0.800667</b> | <b>0.893660</b> | <b>0.688988</b> | <b>0.653838</b> | <b>0.827809</b> |
| $X_1$    | <b>0.810901</b> | 1.000000        | 0.474082        | 0.666741        | 0.645384        | 0.451390        | 0.537709        | 0.597330        |
| $X_2$    | <b>0.776641</b> | 0.474082        | 1.000000        | 0.554616        | 0.688543        | 0.552392        | 0.409121        | 0.569299        |
| $X_3$    | <b>0.800667</b> | 0.666741        | 0.554616        | 1.000000        | 0.731257        | 0.438487        | 0.426919        | 0.491880        |
| $X_4$    | <b>0.893660</b> | 0.645384        | 0.688543        | 0.731257        | 1.000000        | 0.526140        | 0.606229        | 0.708120        |
| $X_5$    | <b>0.688988</b> | 0.451390        | 0.552392        | 0.438487        | 0.526140        | 1.000000        | 0.324583        | 0.630256        |
| $X_6$    | <b>0.653838</b> | 0.537709        | 0.409121        | 0.426919        | 0.606229        | 0.324583        | 1.000000        | 0.608009        |
| $X_7$    | <b>0.827809</b> | 0.597330        | 0.569299        | 0.491880        | 0.708120        | 0.630256        | 0.608009        | 1.000000        |
| Weights  |                 | 0.406915        | 0.347111        | 0.382829        | 0.561247        | 0.295479        | 0.250909        | 0.319554        |
| Loadings |                 | 0.810901        | 0.776641        | 0.800667        | 0.89366         | 0.688988        | 0.653838        | 0.827809        |

| Sl. No.   | Ranking Scores of 30 candidates awarded by Seven Evaluators |       |       |       |       |       |       | Composite Score (Y) Optimized Results |       | Rank-Order ( $Z_2$ ) Optimized Results |       |
|-----------|---|-------|-------|-------|-------|-------|-------|---------------------------------------|-------|--|-------|
|           | $X_1$   | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | Y                                     | $Z_1$ | Y'                                     | $Z_2$ |
| 1         | 6   | 9     | 3     | 12    | 1     | 3     | 11    | 17.70085                              | 2     | 17.40745                               | 2     |
| <b>2</b>  | 25  | 17    | 19    | 23    | 18    | 30    | 7     | 49.33984                              | 20    | 50.79072                               | 21    |
| 3         | 1   | 11    | 6     | 15    | 11    | 29    | 21    | 32.42039                              | 13    | 32.71282                               | 13    |
| 4         | 12  | 15    | 15    | 27    | 9     | 20    | 20    | 44.63941                              | 18    | 44.92521                               | 18    |
| <b>5</b>  | 20  | 26    | 27    | 17    | 15    | 28    | 18    | 53.54956                              | 24    | 54.44562                               | 25    |
| 6         | 8   | 7     | 12    | 4     | 22    | 9     | 14    | 27.13537                              | 6     | 26.65443                               | 6     |
| 7         | 21  | 10    | 24    | 19    | 13    | 19    | 22    | 48.77574                              | 19    | 49.11107                               | 19    |
| 8         | 27  | 14    | 22    | 25    | 28    | 15    | 30    | 61.45677                              | 29    | 60.82086                               | 29    |
| 9         | 24  | 6     | 14    | 10    | 8     | 17    | 13    | 34.06021                              | 14    | 34.68700                               | 14    |
| 10        | 4   | 1     | 11    | 14    | 10    | 8     | 3     | 19.95753                              | 4     | 20.30134                               | 4     |
| 11        | 29  | 25    | 10    | 21    | 7     | 26    | 28    | 52.68741                              | 23    | 52.95060                               | 23    |
| <b>12</b> | 13  | 28    | 17    | 29    | 20    | 10    | 26    | 53.88032                              | 25    | 53.03821                               | 24    |
| <b>13</b> | 23  | 23    | 1     | 9     | 3     | 25    | 10    | 30.50765                              | 11    | 31.42658                               | 12    |
| <b>14</b> | 22  | 5     | 28    | 18    | 5     | 23    | 27    | 49.81567                              | 21    | 50.44069                               | 20    |
| 15        | 10  | 21    | 26    | 26    | 17    | 11    | 25    | 52.34680                              | 22    | 51.87400                               | 22    |
| 16        | 16  | 24    | 21    | 28    | 14    | 16    | 29    | 56.14449                              | 26    | 55.76413                               | 26    |
| 17        | 28  | 22    | 30    | 30    | 30    | 5     | 16    | 62.14490                              | 30    | 61.80191                               | 30    |
| 18        | 3   | 27    | 4     | 6     | 12    | 21    | 15    | 27.59234                              | 8     | 27.59011                               | 8     |
| 19        | 11  | 3     | 16    | 5     | 19    | 24    | 4     | 27.70509                              | 9     | 28.74193                               | 9     |
| 20        | 17  | 13    | 23    | 8     | 29    | 13    | 2     | 36.86148                              | 15    | 37.34405                               | 15    |
| 21        | 19  | 20    | 25    | 22    | 27    | 27    | 23    | 58.98898                              | 28    | 59.27033                               | 28    |
| 22        | 9   | 29    | 13    | 2     | 6     | 4     | 17    | 27.57817                              | 7     | 26.98883                               | 7     |
| 23        | 5   | 30    | 2     | 20    | 26    | 22    | 24    | 43.75418                              | 17    | 43.00494                               | 17    |
| 24        | 18  | 16    | 5     | 11    | 16    | 14    | 6     | 29.43173                              | 10    | 29.71901                               | 10    |
| 25        | 30  | 12    | 29    | 24    | 25    | 7     | 19    | 57.21300                              | 27    | 56.99428                               | 27    |
| 26        | 2   | 19    | 7     | 1     | 4     | 2     | 12    | 15.97747                              | 1     | 15.43028                               | 1     |
| 27        | 14  | 18    | 20    | 13    | 23    | 18    | 8     | 39.98784                              | 16    | 40.46898                               | 16    |
| 28        | 7   | 8     | 8     | 16    | 2     | 6     | 1     | 18.71262                              | 3     | 19.19911                               | 3     |
| 29        | 15  | 2     | 18    | 7     | 21    | 1     | 5     | 26.77338                              | 5     | 26.62086                               | 5     |
| <b>30</b> | 26  | 4     | 9     | 3     | 24    | 12    | 9     | 30.68758                              | 12    | 30.74115                               | 11    |

| <b>Table-2.2: Inter-Correlation Matrix, Weights and Component Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-2 (<math>F_1=2.610741</math>; <math>S_1= 2.656011</math>)</b> |                 |                 |                 |                 |                 |                 |                 |                 |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|   | $Z_1$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
| $Z_1$   | 1.000000        | <b>0.649833</b> | <b>0.389989</b> | <b>0.703226</b> | <b>0.812236</b> | <b>0.510567</b> | <b>0.384650</b> | <b>0.688098</b> |
| $X_1$   | <b>0.649833</b> | 1.000000        | -0.014905       | 0.519021        | 0.355729        | 0.327697        | 0.209344        | 0.205339        |
| $X_2$   | <b>0.389989</b> | -0.014905       | 1.000000        | -0.039822       | 0.314794        | 0.054060        | 0.216463        | 0.471858        |
| $X_3$   | <b>0.703226</b> | 0.519021        | -0.039822       | 1.000000        | 0.511902        | 0.477642        | 0.022914        | 0.305451        |
| $X_4$   | <b>0.812236</b> | 0.355729        | 0.314794        | 0.511902        | 1.000000        | 0.253393        | 0.209789        | 0.607119        |
| $X_5$   | <b>0.510567</b> | 0.327697        | 0.054060        | 0.477642        | 0.253393        | 1.000000        | 0.009121        | 0.070078        |
| $X_6$   | <b>0.384650</b> | 0.209344        | 0.216463        | 0.022914        | 0.209789        | 0.009121        | 1.000000        | 0.277419        |
| $X_7$   | <b>0.688098</b> | 0.205339        | 0.471858        | 0.305451        | 0.607119        | 0.070078        | 0.277419        | 1.000000        |
| Weights   |                 | 0.389271        | 0.245369        | 0.444693        | 0.503508        | 0.309999        | 0.225894        | 0.435731        |
| Loadings  |                 | 0.634388        | 0.399833        | 0.724691        | 0.820595        | 0.505245        | 0.368176        | 0.710153        |

| <b>Table-2.3: Inter-Correlation Matrix, Weights and Component Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-2 (<math>F_2=2.610967</math>; <math>S_2=2.610967</math>)</b> |                 |                 |                 |                 |                 |                 |                 |                 |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|   | $Z_2$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
| $Z_2$   | 1.000000        | <b>0.652948</b> | <b>0.402892</b> | <b>0.700111</b> | <b>0.811791</b> | <b>0.504783</b> | <b>0.401557</b> | <b>0.676085</b> |
| $X_1$   | <b>0.652948</b> | 1.000000        | -0.014905       | 0.519021        | 0.355729        | 0.327697        | 0.209344        | 0.205339        |
| $X_2$   | <b>0.402892</b> | -0.014905       | 1.000000        | -0.039822       | 0.314794        | 0.054060        | 0.216463        | 0.471858        |
| $X_3$   | <b>0.700111</b> | 0.519021        | -0.039822       | 1.000000        | 0.511902        | 0.477642        | 0.022914        | 0.305451        |
| $X_4$   | <b>0.811791</b> | 0.355729        | 0.314794        | 0.511902        | 1.000000        | 0.253393        | 0.209789        | 0.607119        |
| $X_5$   | <b>0.504783</b> | 0.327697        | 0.054060        | 0.477642        | 0.253393        | 1.000000        | 0.009121        | 0.070078        |
| $X_6$   | <b>0.401557</b> | 0.209344        | 0.216463        | 0.022914        | 0.209789        | 0.009121        | 1.000000        | 0.277419        |
| $X_7$   | <b>0.676085</b> | 0.205339        | 0.471858        | 0.305451        | 0.607119        | 0.070078        | 0.277419        | 1.000000        |
| Weights   |                 | 0.401917        | 0.239038        | 0.466325        | 0.507664        | 0.283960        | 0.277857        | 0.385103        |
| Loadings  |                 | 0.652948        | 0.402892        | 0.700111        | 0.811791        | 0.504783        | 0.401557        | 0.676085        |

| <b>Table-3.1: Dataset Relating to Example-3 Showing Sub-optimality of PC-based Rank-ordering of Objects</b> |   |       |       |       |       |       |       |                                       |       |  |       |
|---|---|-------|-------|-------|-------|-------|-------|---------------------------------------|-------|--|-------|
| Sl. No.   | Ranking Scores of 30 candidates awarded by Seven Evaluators |       |       |       |       |       |       | Composite Score (Y) Optimized Results |       | Rank-Order ( $Z_2$ ) Optimized Results |       |
|   | $X_1$   | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | Y                                     | $Z_1$ | Y'                                     | $Z_2$ |
| 1   | 19  | 16    | 14    | 20    | 15    | 1     | 18    | 39.79121                              | 15    | 39.47904                               | 15    |
| 2   | 27  | 18    | 12    | 19    | 10    | 21    | 4     | 41.68016                              | 18    | 41.95384                               | 18    |
| 3   | 21  | 23    | 20    | 21    | 26    | 27    | 29    | 62.78029                              | 25    | 62.64734                               | 25    |
| 4   | 18  | 17    | 13    | 15    | 9     | 13    | 17    | 38.58541                              | 14    | 38.72100                               | 14    |
| 5   | 9   | 9     | 25    | 10    | 20    | 14    | 22    | 41.09776                              | 17    | 41.26278                               | 17    |
| 6   | 20  | 30    | 18    | 23    | 24    | 23    | 20    | 59.43680                              | 23    | 59.13327                               | 23    |
| 7   | 11  | 5     | 6     | 6     | 3     | 7     | 9     | 17.78085                              | 4     | 18.01278                               | 4     |
| 8   | 26  | 27    | 22    | 30    | 25    | 25    | 28    | 69.21167                              | 28    | 68.86556                               | 28    |
| 9   | 23  | 28    | 29    | 28    | 21    | 29    | 30    | 70.78323                              | 30    | 70.76889                               | 30    |
| 10  | 7   | 21    | 8     | 5     | 6     | 4     | 2     | 19.83920                              | 6     | 19.95443                               | 6     |
| 11  | 17  | 15    | 7     | 11    | 13    | 5     | 16    | 32.08370                              | 12    | 32.03662                               | 12    |
| 12  | 22  | 24    | 24    | 13    | 12    | 28    | 12    | 50.14432                              | 20    | 50.90558                               | 20    |
| 13  | 16  | 7     | 1     | 2     | 14    | 3     | 6     | 18.66655                              | 5     | 18.81742                               | 5     |
| 14  | 10  | 1     | 3     | 12    | 22    | 18    | 1     | 25.00260                              | 8     | 24.48451                               | 8     |

|    |    |    |    |    |    |    |    |          |    |          |    |
|----|----|----|----|----|----|----|----|----------|----|----------|----|
| 15 | 13 | 12 | 11 | 22 | 23 | 6  | 13 | 38.42074 | 13 | 37.58991 | 13 |
| 16 | 14 | 19 | 19 | 14 | 19 | 11 | 19 | 43.54547 | 19 | 43.50900 | 19 |
| 17 | 28 | 29 | 30 | 16 | 18 | 12 | 11 | 54.47542 | 22 | 55.15631 | 22 |
| 18 | 3  | 2  | 2  | 9  | 4  | 10 | 14 | 16.58316 | 3  | 16.29347 | 3  |
| 19 | 24 | 22 | 28 | 25 | 29 | 30 | 27 | 69.59663 | 29 | 69.55277 | 29 |
| 20 | 4  | 3  | 17 | 18 | 5  | 8  | 3  | 22.13969 | 7  | 21.83190 | 7  |
| 21 | 25 | 25 | 26 | 24 | 28 | 26 | 25 | 67.44970 | 27 | 67.41736 | 27 |
| 22 | 1  | 4  | 15 | 1  | 7  | 9  | 5  | 15.47967 | 1  | 15.80932 | 2  |
| 23 | 5  | 20 | 5  | 8  | 11 | 17 | 7  | 26.89428 | 11 | 26.71896 | 11 |
| 24 | 29 | 26 | 23 | 26 | 27 | 20 | 26 | 67.11005 | 26 | 66.98988 | 26 |
| 25 | 8  | 6  | 4  | 4  | 16 | 22 | 10 | 25.65787 | 10 | 25.67207 | 10 |
| 26 | 6  | 11 | 9  | 3  | 2  | 2  | 8  | 15.50943 | 2  | 15.78531 | 1  |
| 27 | 30 | 13 | 21 | 27 | 30 | 15 | 23 | 60.76023 | 24 | 60.44323 | 24 |
| 28 | 12 | 10 | 27 | 29 | 17 | 16 | 24 | 51.46019 | 21 | 50.94166 | 21 |
| 29 | 2  | 8  | 16 | 7  | 1  | 19 | 15 | 25.03077 | 9  | 25.36557 | 9  |
| 30 | 15 | 14 | 10 | 17 | 8  | 24 | 21 | 40.77648 | 16 | 40.76065 | 16 |

**Table-3.2: Inter-Correlation Matrix, Weights and Component Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-3 ( $F_1=4.476465$ ;  $S_1= 4.558674$ )**

|          | $Z_1$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_1$    | 1.000000        | <b>0.824249</b> | <b>0.779755</b> | <b>0.798443</b> | <b>0.860289</b> | <b>0.797998</b> | <b>0.716574</b> | <b>0.813126</b> |
| $X_1$    | <b>0.824249</b> | 1.000000        | 0.720133        | 0.555951        | 0.710790        | 0.694327        | 0.453170        | 0.553281        |
| $X_2$    | <b>0.779755</b> | 0.720133        | 1.000000        | 0.600445        | 0.557286        | 0.506563        | 0.494994        | 0.529255        |
| $X_3$    | <b>0.798443</b> | 0.555951        | 0.600445        | 1.000000        | 0.675640        | 0.538598        | 0.509232        | 0.662291        |
| $X_4$    | <b>0.860289</b> | 0.710790        | 0.557286        | 0.675640        | 1.000000        | 0.706785        | 0.517241        | 0.727697        |
| $X_5$    | <b>0.797998</b> | 0.694327        | 0.506563        | 0.538598        | 0.706785        | 1.000000        | 0.492325        | 0.640489        |
| $X_6$    | <b>0.716574</b> | 0.453170        | 0.494994        | 0.509232        | 0.517241        | 0.492325        | 1.000000        | 0.551947        |
| $X_7$    | <b>0.813126</b> | 0.553281        | 0.529255        | 0.662291        | 0.727697        | 0.640489        | 0.551947        | 1.000000        |
| Weights  |                 | 0.391189        | 0.364617        | 0.377348        | 0.409665        | 0.381840        | 0.327169        | 0.388544        |
| Loadings |                 | 0.835132        | 0.778517        | 0.805647        | 0.874764        | 0.815356        | 0.698526        | 0.829528        |

**Table-3.3: Inter-Correlation Matrix, Weights and Component Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-3 ( $F_2=4.476555$ ;  $S_2=4.476555$ )**

|          | $Z_2$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_2$    | 1.000000        | <b>0.822024</b> | <b>0.776641</b> | <b>0.801112</b> | <b>0.859399</b> | <b>0.800222</b> | <b>0.719689</b> | <b>0.811791</b> |
| $X_1$    | <b>0.822024</b> | 1.000000        | 0.720133        | 0.555951        | 0.710790        | 0.694327        | 0.453170        | 0.553281        |
| $X_2$    | <b>0.776641</b> | 0.720133        | 1.000000        | 0.600445        | 0.557286        | 0.506563        | 0.494994        | 0.529255        |
| $X_3$    | <b>0.801112</b> | 0.555951        | 0.600445        | 1.000000        | 0.675640        | 0.538598        | 0.509232        | 0.662291        |
| $X_4$    | <b>0.859399</b> | 0.710790        | 0.557286        | 0.675640        | 1.000000        | 0.706785        | 0.517241        | 0.727697        |
| $X_5$    | <b>0.800222</b> | 0.694327        | 0.506563        | 0.538598        | 0.706785        | 1.000000        | 0.492325        | 0.640489        |
| $X_6$    | <b>0.719689</b> | 0.453170        | 0.494994        | 0.509232        | 0.517241        | 0.492325        | 1.000000        | 0.551947        |
| $X_7$    | <b>0.811791</b> | 0.553281        | 0.529255        | 0.662291        | 0.727697        | 0.640489        | 0.551947        | 1.000000        |
| Weights  |                 | 0.424073        | 0.364980        | 0.405664        | 0.360753        | 0.357984        | 0.337184        | 0.387815        |
| Loadings |                 | 0.822024        | 0.776641        | 0.801112        | 0.859399        | 0.800222        | 0.719689        | 0.811791        |

| Sl. No. | Ranking Scores of 30 candidates awarded by Seven Evaluators |                |                |                |                |                |                | Composite Score (Y'')<br>Optimized Results |                   | Rank-Order (Z <sub>2</sub> '')<br>Optimized Results |                   |
|---------|---|----------------|----------------|----------------|----------------|----------------|----------------|--|-------------------|---|-------------------|
|         | X <sub>1</sub>  | X <sub>2</sub> | X <sub>3</sub> | X <sub>4</sub> | X <sub>5</sub> | X <sub>6</sub> | X <sub>7</sub> | Y''  | Z <sub>1</sub> '' | Y'''  | Z <sub>2</sub> '' |
| 1       | 18  | 18             | 5              | 3              | 13             | 22             | 4              | 31.371434                                  | 10                | 31.62440  | 10                |
| 2       | 20  | 22             | 30             | 27             | 7              | 13             | 13             | 49.890877                                  | 22                | 50.03646  | 22                |
| 3       | 3   | 29             | 20             | 20             | 21             | 11             | 18             | 46.111595                                  | 19                | 46.88164  | 19                |
| 4       | 10  | 27             | 7              | 13             | 25             | 8              | 23             | 42.710587                                  | 18                | 43.15021  | 18                |
| 5       | 9   | 11             | 10             | 7              | 5              | 24             | 15             | 30.614628                                  | 8                 | 30.17925  | 8                 |
| 6       | 12  | 8              | 26             | 17             | 16             | 16             | 14             | 41.197770                                  | 15                | 40.99650  | 15                |
| 7       | 7   | 14             | 9              | 8              | 28             | 26             | 1              | 35.150913                                  | 13                | 35.71881  | 13                |
| 8       | 6   | 13             | 12             | 14             | 15             | 28             | 24             | 42.331403                                  | 17                | 41.78964  | 17                |
| 9       | 27  | 17             | 27             | 22             | 19             | 18             | 28             | 59.718366                                  | 24                | 59.15019  | 24                |
| 10      | 15  | 28             | 21             | 19             | 29             | 27             | 29             | 63.498033                                  | 29                | 63.56338  | 29                |
| 11      | 14  | 25             | 23             | 11             | 17             | 19             | 17             | 47.623564                                  | 20                | 47.99190  | 21                |
| 12      | 17  | 19             | 19             | 12             | 22             | 7              | 30             | 47.623968                                  | 21                | 47.44355  | 20                |
| 13      | 4   | 1              | 8              | 5              | 20             | 12             | 7              | 21.544108                                  | 5                 | 21.47765  | 5                 |
| 14      | 16  | 12             | 11             | 4              | 9              | 14             | 8              | 27.969578                                  | 7                 | 27.95407  | 7                 |
| 15      | 25  | 24             | 28             | 16             | 23             | 20             | 25             | 60.852521                                  | 26                | 60.81207  | 26                |
| 16      | 2   | 6              | 18             | 23             | 8              | 21             | 11             | 33.637782                                  | 11                | 33.29539  | 11                |
| 17      | 29  | 20             | 24             | 18             | 27             | 29             | 20             | 63.120288                                  | 28                | 62.91071  | 28                |
| 18      | 22  | 9              | 3              | 10             | 1              | 3              | 19             | 25.323880                                  | 6                 | 24.59793  | 6                 |
| 19      | 24  | 23             | 22             | 15             | 24             | 30             | 26             | 61.986236                                  | 27                | 61.71055  | 27                |
| 20      | 19  | 5              | 17             | 24             | 14             | 5              | 9              | 35.150701                                  | 12                | 34.87592  | 12                |
| 21      | 5   | 2              | 1              | 6              | 3              | 6              | 22             | 17.008182                                  | 3                 | 16.16041  | 3                 |
| 22      | 8   | 10             | 13             | 1              | 2              | 1              | 5              | 15.118738                                  | 2                 | 15.36358  | 2                 |
| 23      | 26  | 26             | 25             | 25             | 30             | 17             | 10             | 60.096859                                  | 25                | 60.72466  | 25                |
| 24      | 13  | 4              | 4              | 9              | 10             | 9              | 6              | 20.788207                                  | 4                 | 20.54303  | 4                 |
| 25      | 21  | 21             | 16             | 30             | 18             | 23             | 27             | 58.962115                                  | 23                | 58.44723  | 23                |
| 26      | 30  | 15             | 14             | 21             | 12             | 2              | 16             | 41.576617                                  | 16                | 41.30021  | 16                |
| 27      | 11  | 16             | 15             | 29             | 4              | 15             | 12             | 38.551624                                  | 14                | 38.39481  | 14                |
| 28      | 23  | 7              | 6              | 26             | 6              | 10             | 3              | 30.615030                                  | 9                 | 30.24845  | 9                 |
| 29      | 1   | 3              | 2              | 2              | 11             | 4              | 2              | 9.449315                                   | 1                 | 9.62243   | 1                 |
| 30      | 28  | 30             | 29             | 28             | 26             | 25             | 21             | 70.679438                                  | 30                | 70.88402  | 30                |

|                | Z <sub>1</sub>  | X <sub>1</sub>  | X <sub>2</sub>  | X <sub>3</sub>  | X <sub>4</sub>  | X <sub>5</sub>  | X <sub>6</sub>  | X <sub>7</sub>  |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Z <sub>1</sub> | 1.000000        | <b>0.594661</b> | <b>0.846496</b> | <b>0.820690</b> | <b>0.624027</b> | <b>0.738821</b> | <b>0.598220</b> | <b>0.671190</b> |
| X <sub>1</sub> | <b>0.594661</b> | 1.000000        | 0.426474        | 0.480311        | 0.478977        | 0.277864        | 0.164850        | 0.321913        |
| X <sub>2</sub> | <b>0.846496</b> | 0.426474        | 1.000000        | 0.629366        | 0.410456        | 0.638265        | 0.441602        | 0.532369        |
| X <sub>3</sub> | <b>0.820690</b> | 0.480311        | 0.629366        | 1.000000        | 0.596885        | 0.490545        | 0.422469        | 0.441602        |
| X <sub>4</sub> | <b>0.624027</b> | 0.478977        | 0.410456        | 0.596885        | 1.000000        | 0.213793        | 0.220022        | 0.309900        |
| X <sub>5</sub> | <b>0.738821</b> | 0.277864        | 0.638265        | 0.490545        | 0.213793        | 1.000000        | 0.519911        | 0.369077        |
| X <sub>6</sub> | <b>0.598220</b> | 0.164850        | 0.441602        | 0.422469        | 0.220022        | 0.519911        | 1.000000        | 0.302336        |
| X <sub>7</sub> | <b>0.671190</b> | 0.321913        | 0.532369        | 0.441602        | 0.309900        | 0.369077        | 0.302336        | 1.000000        |
| Weights        |                 | 0.377992        | 0.377978        | 0.377951        | 0.377940        | 0.377995        | 0.377938        | 0.377957        |
| Loadings       |                 | 0.638074        | 0.826052        | 0.822529        | 0.654188        | 0.710798        | 0.622017        | 0.663747        |

|          | $Z_2$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_2$    | 1.000000        | <b>0.593326</b> | <b>0.849166</b> | <b>0.822469</b> | <b>0.623582</b> | <b>0.736596</b> | <b>0.603560</b> | <b>0.665406</b> |
| $X_1$    | <b>0.593326</b> | 1.000000        | 0.426474        | 0.480311        | 0.478977        | 0.277864        | 0.164850        | 0.321913        |
| $X_2$    | <b>0.849166</b> | 0.426474        | 1.000000        | 0.629366        | 0.410456        | 0.638265        | 0.441602        | 0.532369        |
| $X_3$    | <b>0.822469</b> | 0.480311        | 0.629366        | 1.000000        | 0.596885        | 0.490545        | 0.422469        | 0.441602        |
| $X_4$    | <b>0.623582</b> | 0.478977        | 0.410456        | 0.596885        | 1.000000        | 0.213793        | 0.220022        | 0.309900        |
| $X_5$    | <b>0.736596</b> | 0.277864        | 0.638265        | 0.490545        | 0.213793        | 1.000000        | 0.519911        | 0.369077        |
| $X_6$    | <b>0.603560</b> | 0.164850        | 0.441602        | 0.422469        | 0.220022        | 0.519911        | 1.000000        | 0.302336        |
| $X_7$    | <b>0.665406</b> | 0.321913        | 0.532369        | 0.441602        | 0.309900        | 0.369077        | 0.302336        | 1.000000        |
| Weights  |                 | 0.361685        | 0.420167        | 0.387054        | 0.370485        | 0.395085        | 0.363555        | 0.342504        |
| Loadings |                 | 0.593326        | 0.849166        | 0.822469        | 0.623582        | 0.736596        | 0.603560        | 0.665406        |

| Sl. No.   | Ranking Scores of 30 candidates awarded by Seven Evaluators |       |       |       |       |       |       | Composite Score ( $Y''$ ) Optimized Results |         | Rank-Order ( $Z_2''$ ) Optimized Results |         |
|-----------|---|-------|-------|-------|-------|-------|-------|---|---------|--|---------|
|           | $X_1$   | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $Y''$                                       | $Z_1''$ | $Y'''$                                   | $Z_2''$ |
| 1         | 20  | 30    | 18    | 14    | 12    | 9     | 14    | 44.22174                                    | 18      | 44.08011                                 | 18      |
| 2         | 1   | 12    | 4     | 17    | 3     | 5     | 6     | 18.14242                                    | 1       | 17.92374                                 | 1       |
| 3         | 26  | 28    | 26    | 7     | 27    | 30    | 29    | 65.38783                                    | 29      | 65.07686                                 | 29      |
| 4         | 27  | 19    | 6     | 10    | 11    | 16    | 20    | 41.19832                                    | 17      | 41.17978                                 | 17      |
| 5         | 3   | 14    | 9     | 3     | 4     | 15    | 1     | 18.52015                                    | 2       | 18.71988                                 | 2       |
| 6         | 7   | 17    | 2     | 5     | 2     | 19    | 2     | 20.41032                                    | 4       | 20.59862                                 | 4       |
| 7         | 4   | 11    | 27    | 16    | 28    | 18    | 26    | 49.13530                                    | 21      | 48.30841                                 | 21      |
| 8         | 30  | 25    | 28    | 28    | 19    | 10    | 30    | 64.25320                                    | 28      | 64.26996                                 | 28      |
| <b>9</b>  | 17  | 4     | 1     | 25    | 6     | 6     | 18    | 29.10329                                    | 9       | 29.06091                                 | 8       |
| 10        | 25  | 8     | 24    | 21    | 18    | 26    | 19    | 53.29222                                    | 22      | 53.89539                                 | 22      |
| 11        | 6   | 9     | 7     | 4     | 8     | 13    | 12    | 22.29989                                    | 5       | 22.18983                                 | 5       |
| 12        | 11  | 18    | 30    | 20    | 25    | 27    | 27    | 59.71790                                    | 26      | 59.45911                                 | 26      |
| 13        | 14  | 20    | 16    | 6     | 13    | 12    | 16    | 36.66241                                    | 13      | 36.48242                                 | 13      |
| 14        | 8   | 29    | 20    | 30    | 30    | 23    | 17    | 59.34161                                    | 25      | 57.99207                                 | 25      |
| 15        | 22  | 23    | 10    | 29    | 17    | 24    | 25    | 56.69511                                    | 24      | 56.32758                                 | 24      |
| <b>16</b> | 16  | 15    | 5     | 11    | 1     | 14    | 15    | 29.10285                                    | 8       | 29.54333                                 | 9       |
| 17        | 5   | 6     | 12    | 9     | 10    | 8     | 4     | 20.41002                                    | 3       | 20.32964                                 | 3       |
| 18        | 2   | 1     | 8     | 15    | 5     | 28    | 5     | 24.18937                                    | 7       | 24.78023                                 | 7       |
| 19        | 12  | 24    | 21    | 18    | 29    | 21    | 24    | 56.31741                                    | 23      | 55.20101                                 | 23      |
| 20        | 23  | 16    | 3     | 12    | 9     | 22    | 11    | 36.28498                                    | 12      | 36.44815                                 | 12      |
| 21        | 24  | 7     | 11    | 22    | 16    | 25    | 21    | 47.62348                                    | 19      | 47.80678                                 | 19      |
| 22        | 28  | 26    | 13    | 19    | 26    | 20    | 28    | 60.47520                                    | 27      | 59.57479                                 | 27      |
| 23        | 13  | 22    | 17    | 8     | 7     | 4     | 9     | 30.23671                                    | 10      | 30.32081                                 | 10      |
| 24        | 21  | 13    | 19    | 24    | 21    | 17    | 13    | 48.37962                                    | 20      | 48.19456                                 | 20      |
| 25        | 9   | 21    | 23    | 2     | 20    | 3     | 22    | 37.79632                                    | 14      | 36.99930                                 | 14      |
| 26        | 19  | 10    | 25    | 13    | 22    | 11    | 8     | 40.82002                                    | 16      | 40.72036                                 | 16      |
| 27        | 18  | 2     | 15    | 1     | 23    | 1     | 3     | 23.81236                                    | 6       | 23.19690                                 | 6       |
| 28        | 10  | 5     | 22    | 27    | 24    | 7     | 7     | 38.55266                                    | 15      | 37.96188                                 | 15      |
| 29        | 15  | 3     | 14    | 26    | 14    | 2     | 10    | 31.74894                                    | 11      | 31.58722                                 | 11      |
| 30        | 29  | 27    | 29    | 23    | 15    | 29    | 23    | 66.14273                                    | 30      | 66.91077                                 | 30      |

**Table-5.2: Inter-Correlation Matrix, Weights and Component Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-5**  
( $AF_1=4.648943$ ;  $AS_1=4.66456597$ ;  $F_1=3.175427$ )

|          | $Z_1$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_1$    | 1.000000        | <b>0.644049</b> | <b>0.603560</b> | <b>0.703226</b> | <b>0.544383</b> | <b>0.739711</b> | <b>0.560845</b> | <b>0.853170</b> |
| $X_1$    | <b>0.644049</b> | 1.000000        | 0.362403        | 0.233370        | 0.296552        | 0.260512        | 0.261846        | 0.548832        |
| $X_2$    | <b>0.603560</b> | 0.362403        | 1.000000        | 0.333482        | 0.057175        | 0.278309        | 0.305451        | 0.555506        |
| $X_3$    | <b>0.703226</b> | 0.233370        | 0.333482        | 1.000000        | 0.240044        | 0.749944        | 0.185762        | 0.497664        |
| $X_4$    | <b>0.544383</b> | 0.296552        | 0.057175        | 0.240044        | 1.000000        | 0.338821        | 0.241824        | 0.390434        |
| $X_5$    | <b>0.739711</b> | 0.260512        | 0.278309        | 0.749944        | 0.338821        | 1.000000        | 0.236930        | 0.562625        |
| $X_6$    | <b>0.560845</b> | 0.261846        | 0.305451        | 0.185762        | 0.241824        | 0.236930        | 1.000000        | 0.441602        |
| $X_7$    | <b>0.853170</b> | 0.548832        | 0.555506        | 0.497664        | 0.390434        | 0.562625        | 0.441602        | 1.000000        |
| Weights  |                 | 0.377957        | 0.377988        | 0.377893        | 0.377968        | 0.378045        | 0.377960        | 0.377942        |
| Loadings |                 | 0.635321        | 0.620066        | 0.694647        | 0.549860        | 0.734729        | 0.573131        | 0.856811        |

**Table-5.3: Inter-Correlation Matrix, Weights and Component Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-5**  
( $AF_2=4.648943$ ;  $AS_2=4.648943$ ;  $F_2=3.159146$ )

|          | $Z_2$           | $X_1$           | $X_2$           | $X_3$           | $X_4$           | $X_5$           | $X_6$           | $X_7$           |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Z_2$    | 1.000000        | <b>0.643604</b> | <b>0.608454</b> | <b>0.705006</b> | <b>0.538154</b> | <b>0.737486</b> | <b>0.564405</b> | <b>0.851835</b> |
| $X_1$    | <b>0.643604</b> | 1.000000        | 0.362403        | 0.233370        | 0.296552        | 0.260512        | 0.261846        | 0.548832        |
| $X_2$    | <b>0.608454</b> | 0.362403        | 1.000000        | 0.333482        | 0.057175        | 0.278309        | 0.305451        | 0.555506        |
| $X_3$    | <b>0.705006</b> | 0.233370        | 0.333482        | 1.000000        | 0.240044        | 0.749944        | 0.185762        | 0.497664        |
| $X_4$    | <b>0.538154</b> | 0.296552        | 0.057175        | 0.240044        | 1.000000        | 0.338821        | 0.241824        | 0.390434        |
| $X_5$    | <b>0.737486</b> | 0.260512        | 0.278309        | 0.749944        | 0.338821        | 1.000000        | 0.236930        | 0.562625        |
| $X_6$    | <b>0.564405</b> | 0.261846        | 0.305451        | 0.185762        | 0.241824        | 0.236930        | 1.000000        | 0.441602        |
| $X_7$    | <b>0.851835</b> | 0.548832        | 0.555506        | 0.497664        | 0.390434        | 0.562625        | 0.441602        | 1.000000        |
| Weights  |                 | 0.403896        | 0.357386        | 0.417812        | 0.377040        | 0.306707        | 0.401047        | 0.370822        |
| Loadings |                 | 0.643604        | 0.608454        | 0.705006        | 0.538154        | 0.737486        | 0.564405        | 0.851835        |

**Table-6.1: Dataset Relating to Example-6 Showing Rank-ordering by Maximization of Minimal Absolute Correlation**

| Sl. No.   | Ranking Scores of 30 candidates awarded by Seven Evaluators |       |       |       |       |       |       | Composite Score ( $Y'$ ) Optimized Results |         | Rank-Order ( $Z_2'$ ) Optimized Results |         |
|-----------|---|-------|-------|-------|-------|-------|-------|--|---------|---|---------|
|           | $X_1$   | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $Y^*$                                      | $Z_1^*$ | $Y^{**}$                                | $Z_2^*$ |
| 1         | 12  | 2     | 9     | 14    | 23    | 3     | 15    | 25.30634                                   | 11      | 25.88073                                | 11      |
| 2         | 19  | 9     | 21    | 28    | 24    | 13    | 11    | 39.49625                                   | 20      | 38.73892                                | 20      |
| 3         | 7   | 21    | 16    | 18    | 6     | 14    | 23    | 29.96293                                   | 14      | 31.40229                                | 14      |
| <b>4</b>  | 24  | 28    | 28    | 22    | 13    | 26    | 20    | 47.93919                                   | 27      | 48.81581                                | 26      |
| 5         | 13  | 3     | 17    | 5     | 10    | 9     | 3     | 17.77573                                   | 4       | 17.45455                                | 4       |
| 6         | 4   | 15    | 14    | 1     | 16    | 19    | 2     | 25.24309                                   | 10      | 25.82015                                | 10      |
| 7         | 20  | 30    | 29    | 20    | 12    | 23    | 22    | 47.22933                                   | 25      | 48.32437                                | 25      |
| <b>8</b>  | 22  | 18    | 24    | 25    | 18    | 20    | 19    | 43.30421                                   | 22      | 43.75467                                | 21      |
| 9         | 9   | 24    | 22    | 17    | 22    | 15    | 30    | 43.68190                                   | 23      | 45.84717                                | 23      |
| <b>10</b> | 28  | 14    | 30    | 2     | 26    | 16    | 27    | 42.69905                                   | 21      | 44.95308                                | 22      |
| <b>11</b> | 1   | 23    | 7     | 9     | 1     | 1     | 10    | 20.65061                                   | 6       | 21.18701                                | 5       |
| 12        | 2   | 5     | 11    | 11    | 20    | 6     | 7     | 22.14594                                   | 8       | 22.18122                                | 8       |
| 13        | 11  | 27    | 12    | 3     | 11    | 11    | 18    | 32.50658                                   | 16      | 34.47639                                | 16      |

|    |    |    |    |    |    |    |    |          |    |          |    |
|----|----|----|----|----|----|----|----|----------|----|----------|----|
| 14 | 25 | 20 | 13 | 10 | 7  | 30 | 17 | 32.36392 | 15 | 34.27532 | 15 |
| 15 | 27 | 25 | 23 | 30 | 25 | 21 | 26 | 54.91396 | 29 | 55.97770 | 29 |
| 16 | 23 | 17 | 25 | 24 | 8  | 24 | 16 | 36.98198 | 19 | 37.19732 | 19 |
| 17 | 14 | 6  | 10 | 8  | 17 | 2  | 12 | 23.33585 | 9  | 23.87632 | 9  |
| 18 | 6  | 12 | 8  | 4  | 4  | 17 | 5  | 16.01656 | 3  | 16.75380 | 3  |
| 19 | 21 | 10 | 15 | 12 | 14 | 10 | 1  | 28.24829 | 13 | 27.48070 | 13 |
| 20 | 26 | 8  | 26 | 27 | 15 | 12 | 14 | 36.53673 | 18 | 35.78311 | 18 |
| 21 | 16 | 11 | 6  | 23 | 21 | 18 | 25 | 33.92366 | 17 | 35.75911 | 17 |
| 22 | 15 | 22 | 19 | 21 | 27 | 28 | 28 | 47.80047 | 26 | 50.10524 | 27 |
| 23 | 8  | 16 | 18 | 19 | 5  | 8  | 13 | 26.38564 | 12 | 26.33481 | 12 |
| 24 | 30 | 13 | 27 | 13 | 28 | 29 | 24 | 46.41054 | 24 | 48.29007 | 24 |
| 25 | 10 | 1  | 1  | 7  | 9  | 5  | 9  | 11.60210 | 1  | 12.27820 | 1  |
| 26 | 3  | 19 | 3  | 15 | 2  | 22 | 8  | 20.61575 | 5  | 21.48559 | 7  |
| 27 | 17 | 29 | 4  | 26 | 30 | 25 | 29 | 52.11630 | 28 | 54.82387 | 28 |
| 28 | 18 | 7  | 2  | 6  | 3  | 4  | 4  | 13.83704 | 2  | 13.95344 | 2  |
| 29 | 29 | 26 | 20 | 29 | 29 | 27 | 21 | 57.47630 | 30 | 58.46160 | 30 |
| 30 | 5  | 4  | 5  | 16 | 19 | 7  | 6  | 21.52460 | 7  | 21.39735 | 6  |

**Table-6.2: Inter-Correlation Matrix, Weights and Component Loadings of Composite Score Optimized Overall Ranking Scores for the Dataset in Example-6**  
(Maximin Correlation=0.671190; Maximum Correlation=0.813126)

|                | Z <sub>1</sub>  | X <sub>1</sub>  | X <sub>2</sub>  | X <sub>3</sub>  | X <sub>4</sub>  | X <sub>5</sub>  | X <sub>6</sub>  | X <sub>7</sub>  |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Z <sub>1</sub> | 1.000000        | <b>0.676085</b> | <b>0.673415</b> | <b>0.690323</b> | <b>0.684538</b> | <b>0.671190</b> | <b>0.700111</b> | <b>0.813126</b> |
| X <sub>1</sub> | <b>0.676085</b> | 1.000000        | 0.206229        | 0.636485        | 0.423359        | 0.442047        | 0.534594        | 0.451835        |
| X <sub>2</sub> | <b>0.673415</b> | 0.206229        | 1.000000        | 0.353949        | 0.360178        | 0.070523        | 0.624472        | 0.614683        |
| X <sub>3</sub> | <b>0.690323</b> | 0.636485        | 0.353949        | 1.000000        | 0.362848        | 0.324583        | 0.461624        | 0.454505        |
| X <sub>4</sub> | <b>0.684538</b> | 0.423359        | 0.360178        | 0.362848        | 1.000000        | 0.403337        | 0.418465        | 0.493660        |
| X <sub>5</sub> | <b>0.671190</b> | 0.442047        | 0.070523        | 0.324583        | 0.403337        | 1.000000        | 0.318354        | 0.552836        |
| X <sub>6</sub> | <b>0.700111</b> | 0.534594        | 0.624472        | 0.461624        | 0.418465        | 0.318354        | 1.000000        | 0.550612        |
| X <sub>7</sub> | <b>0.813126</b> | 0.451835        | 0.614683        | 0.454505        | 0.493660        | 0.552836        | 0.550612        | 1.000000        |
| Weights        |                 | 0.276166        | 0.652291        | 0.256872        | 0.269181        | 0.595985        | 0.044783        | 0.051025        |
| Loadings       |                 | 0.680300        | 0.680222        | 0.680243        | 0.680277        | 0.680222        | 0.714510        | 0.804249        |

**Table-6.3: Inter-Correlation Matrix, Weights and Component Loadings of Rank Order Optimized Overall Ranking Scores for the Dataset in Example-6**  
(Maximin Correlation=0.673860; Maximum Correlation=0.820245)

|                | Z <sub>2</sub>  | X <sub>1</sub>  | X <sub>2</sub>  | X <sub>3</sub>  | X <sub>4</sub>  | X <sub>5</sub>  | X <sub>6</sub>  | X <sub>7</sub>  |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Z <sub>2</sub> | 1.000000        | <b>0.674750</b> | <b>0.673860</b> | <b>0.686318</b> | <b>0.676085</b> | <b>0.673860</b> | <b>0.715239</b> | <b>0.820245</b> |
| X <sub>1</sub> | <b>0.674750</b> | 1.000000        | 0.206229        | 0.636485        | 0.423359        | 0.442047        | 0.534594        | 0.451835        |
| X <sub>2</sub> | <b>0.673860</b> | 0.206229        | 1.000000        | 0.353949        | 0.360178        | 0.070523        | 0.624472        | 0.614683        |
| X <sub>3</sub> | <b>0.686318</b> | 0.636485        | 0.353949        | 1.000000        | 0.362848        | 0.324583        | 0.461624        | 0.454505        |
| X <sub>4</sub> | <b>0.676085</b> | 0.423359        | 0.360178        | 0.362848        | 1.000000        | 0.403337        | 0.418465        | 0.493660        |
| X <sub>5</sub> | <b>0.673860</b> | 0.442047        | 0.070523        | 0.324583        | 0.403337        | 1.000000        | 0.318354        | 0.552836        |
| X <sub>6</sub> | <b>0.715239</b> | 0.534594        | 0.624472        | 0.461624        | 0.418465        | 0.318354        | 1.000000        | 0.550612        |
| X <sub>7</sub> | <b>0.820245</b> | 0.451835        | 0.614683        | 0.454505        | 0.493660        | 0.552836        | 0.550612        | 1.000000        |
| Weights        |                 | 0.269713        | 0.664980        | 0.215650        | 0.208353        | 0.603497        | 0.082781        | 0.155176        |
| Loadings       |                 | 0.674750        | 0.673860        | 0.686318        | 0.676085        | 0.673860        | 0.715239        | 0.820245        |

```

1: C      MAIN PROGRAM : PROVIDES TO USE REPULSIVE PARTICLE SWARM METHOD TO
2: C                      COMPUTE COMPOSITE INDEX INDICES
3: C      BY MAXIMIZING SUM OF (SQUARES, OR ABSOLUTES, OR MINIMUM) OF
4: C      CORRELATION OF THE INDEX WITH THE CONSTITUENT VARIABLES. THE MAX
5: C      SUM OF SQUARES IS THE PRINCIPAL COMPONENT INDEX. IT ALSO PROVIDES
6: C      TO OBTAIN MAXIMUM ENTROPY ABSOLUTE CORRELATION INDICES.
7: C      PRODUCT MOMENT AS WELL AS ABSOLUTE CORRELATION (BRADLEY, 1985) MAY
8: C      BE USED. PROGRAM BY SK MISHRA, DEPT. OF ECONOMICS, NORTH-EASTERN
9: C      HILL UNIVERSITY, SHILLONG (INDIA)
10: C      -----
11: C      ADJUST THE PARAMETERS SUITABLY IN SUBROUTINES RPS
12: C      WHEN THE PROGRAM ASKS FOR PARAMETERS, FEED THEM SUITABLY
13: C      -----
14: C      PROGRAM RPSINDEX
15: C      PARAMETER (NOB=30,MVAR=5)!CHANGE THE PARAMETERS HERE AS NEEDED.
16: C      -----
17: C      NOB=NO. OF CASES AND MVAR=NO. OF VARIABLES
18: C      TO BE ADJUSTED IN SUBROUTINE CORD(M,X,F) ALSO: STATEMENT 931
19: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
20: C      COMMON /KFF/KF,NFCALL,FTIT ! FUNCTION CODE, NO. OF CALLS & TITLE
21: C      CHARACTER *30 METHOD(1)
22: C      CHARACTER *70 FTIT
23: C      CHARACTER *40 INFILE,OUTFILE
24: C      COMMON /CORDAT/CDAT (NOB,MVAR) , QIND (NOB) , R (MVAR) , ENTROPY,NORM,NCOR
25: C      COMMON /XBASE/XBAS
26: C      COMMON /RNDM/IU,IV ! RANDOM NUMBER GENERATION (IU = 4-DIGIT SEED)
27: C      COMMON /GETRANK/MRNK
28: C      INTEGER IU,IV
29: C      DIMENSION XX(3,50) , KKF(3) , MM(3) , FMINN(3) , XBAS(1000,50)
30: C      DIMENSION ZDAT (NOB,MVAR+1) , FRANK (NOB) , RMAT (MVAR+1,MVAR+1)
31: C      DIMENSION X(50)! X IS THE DECISION VARIABLE X IN F(X) TO MINIMIZE
32: C      M IS THE DIMENSION OF THE PROBLEM, KF IS TEST FUNCTION CODE AND
33: C      FMIN IS THE MIN VALUE OF F(X) OBTAINED FROM RPS
34: C      WRITE (*,*) '===== WARNING ===== '
35: C      WRITE (*,*) 'ADJUST PARAMETERS IN SUBROUTINES RPS IF NEEDED '
36: C      NOPT=1 ! OPTIMIZATION BY RPS METHOD
37: C      WRITE (*,*) '===== '
38: C      METHOD(1)=' : REPULSIVE PARTICLE SWARM OPTIMIZATION'
39: C      INITIALIZATION. THIS XBAS WILL BE USED TO
40: C      INITIALIZE THE POPULATION.
41: C      WRITE (*,*) ' '
42: C      WRITE (*,*) 'FEED RANDOM NUMBER SEED,NORM,ENTROPY,NCOR'
43: C      WRITE (*,*) 'SEED[ANY 4-DIGIT NUMBER]; NORM[1,2,3]; ENTROPY[0,1]; &
44: C      &NCOR[0,1] '
45: C      WRITE (*,*) ' '
46: C      WRITE (*,*) 'NORM(1)=ABSOLUTE;NORM(2)=PCA-EUCLIDEAN;NORM(3)=MAXIMIN'
47: C      WRITE (*,*) 'ENTROPY(0)=MAXIMIZES NORM;ENTROPY(1)=MAXIMIZES ENTROPY'
48: C      WRITE (*,*) 'NCOR(0)=PRODUCT MOMENT; NCOR(1)=ABSOLUTE CORRELATION'
49: C      READ (*,*) IU,NORM,ENTROPY,NCOR
50: C      WRITE (*,*) 'WANT RANK SCORE OPTIMIZATION? YES(1); NO(OTHER THAN 1)'
51: C      READ (*,*) MRNK
52: C      WRITE (*,*) 'INPUT FILE TO READ DATA:YOUR DATA MUST BE IN THIS FILE'
53: C      WRITE (*,*) 'CASES (NOB) IN ROWS ; VARIABLES (MVAR) IN COLUMNS'
54: C      READ (*,*) INFILE
55: C      WRITE (*,*) 'SPECIFY THE OUTPUT FILE TO STORE THE RESULTS'
56: C      READ (*,*) OUTFILE
57: C      OPEN (9, FILE=OUTFILE)
58: C      OPEN (7, FILE=INFILE)
59: C      DO I=1,NOB
60: C      READ (7,*) ,CDA, (CDAT (I,J) ,J=1,MVAR)
61: C      ENDDO
62: C      CLOSE (7)
63: C      DO I=1,NOB
64: C      DO J=1,MVAR
65: C      ZDAT (I,J+1)=CDAT (I,J)
66: C      ENDDO
67: C      ENDDO

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68:      WRITE(*,*) 'DATA HAS BEEN READ. WOULD YOU UNITIZE VARIABLES? [YES=1
69: & ELSE NO UNITIZATION] '
70:      WRITE(*,*) 'UNITIZE MEANS TRANSFORMATION FROM X(I,J) TO UNITIZED X'
71:      WRITE(*,*) '[X(I,J)-MIN(X(.,J))]/[MAX(X(.,J))-MIN(X(.,J))]'
72:      READ(*,*) NUN
73:      IF(NUN.EQ.1) THEN
74:      DO J=1,MVAR
75:      CMIN=CDAT(1,J)
76:      CMAX=CDAT(1,J)
77:      DO I=2,NOB
78:      IF(CMIN.GT.CDAT(I,J)) CMIN=CDAT(I,J)
79:      IF(CMAX.LT.CDAT(I,J)) CMAX=CDAT(I,J)
80:      ENDDO
81:      DO I=1,NOB
82:      CDAT(I,J)=(CDAT(I,J)-CMIN)/(CMAX-CMIN)
83:      ENDDO
84:      ENDDO
85:      ENDIF
86: C -----
87:      WRITE(*,*) ' '
88:      WRITE(*,*) 'FEED RANDOM NUMBER SEED [4-DIGIT ODD INTEGER] TO BEGIN'
89:      READ(*,*) IU
90: C THIS XBAS WILL BE USED AS INITIAL X
91:      DO I=1,1000
92:      DO J=1,50
93:      CALL RANDOM(RAND)
94:      XBAS(I,J)=RAND ! RANDOM NUMBER BETWEEN (0, 1)
95:      ENDDO
96:      ENDDO
97: C -----
98: C HOWEVER, THE FIRST ROW OF, THAT IS, XBAS(1,J),J=1,MVAR) MAY BE
99: C SPECIFIED HERE IF THE USER KNOWS IT TO BE OPTIMAL OR NEAR-OPTIMAL
100: C DATA (XBAS(1,J),J=1,MVAR) /DATA1, DATA2, ....., DATAMVAR/
101: C -----
102:      WRITE(*,*) ' *****'
103: C -----
104:      DO I=1,NOPT
105:      IF(I.EQ.1) THEN
106:      WRITE(*,*) '==== WELCOME TO RPS PROGRAM FOR INDEX CONSTRUCTION'
107:      CALL RPS(M,X,FMINRPS,Q1) !CALLS RPS AND RETURNS OPTIMAL X AND FMIN
108:      WRITE(*,*) 'RPS BRINGS THE FOLLOWING VALUES TO THE MAIN PROGRAM'
109:      WRITE(*,*) (X(JOPT),JOPT=1,M), ' OPTIMUM FUNCTION=',FMINRPS
110:      IF(KF.EQ.1) THEN
111:      WRITE(9,*) 'PARTICLE SWARM OPTIMIZATION RESULTS'
112:      RSUM1=0.D0
113:      RSUM2=0.D0
114:      DO J=1,MVAR
115:      RSUM1=RSUM1+DABS(R(J))
116:      RSUM2=RSUM2+DABS(R(J))**2
117:      ENDDO
118:      WRITE(9,*) 'CORRELATION OF INDEX WITH CONSTITUENT VARIABLES'
119:      WRITE(9,*) (R(J),J=1,MVAR)
120:      WRITE(9,*) 'SUM OF ABS (R)=' ,RSUM1, '; SUM OF SQUARE(R)=' ,RSUM2
121:      WRITE(9,*) 'THE INDEX OR SCORE OF DIFFERENT CASES'
122:      DO II=1,NOB
123:      WRITE(9,*) QIND(II)
124:      FRANK(II)=QIND(II)
125:      ENDDO
126:      ENDIF
127:      FMIN=FMINRPS
128:      ENDIF
129: C -----
130:      DO J=1,M
131:      XX(I,J)=X(J)
132:      ENDDO
133:      KKF(I)=KF
134:      MM(I)=M

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135:     FMINN(I)=FMIN
136:     ENDDO
137:     WRITE(*,*) ' '
138:     WRITE(*,*) ' '
139:     WRITE(*,*) '----- FINAL RESULTS-----'
140:     DO I=1,NOPT
141:     WRITE(*,*) 'FUNCT CODE=',KKF(I),' FMIN=',FMINN(I),' : DIM=',MM(I)
142:     WRITE(*,*) 'OPTIMAL DECISION VARIABLES : ',METHOD(I)
143:     WRITE(*,*) 'WEIGHTS ARE AS FOLLOWS -----'
144:     WRITE(9,*) 'WEIGHTS ARE AS FOLLOWS -----'
145:     WRITE(9,*) (XX(I,J),J=1,M)
146:     WRITE(*,*) (XX(I,J),J=1,M)
147:     WRITE(*,*) '/////////////////////////////////////'
148:     ENDDO
149:     WRITE(*,*) 'OPTIMIZATION PROGRAM ENDED'
150:     WRITE(*,*) '*****'
151:     WRITE(*,*) 'MEASURE OF EQUALITY/INEQUALITY'
152:     WRITE(*,*) 'RPS: BEFORE AND AFTER OPTIMIZATION = ',Q0,Q1
153:     WRITE(*,*) ' '
154:     WRITE(*,*) 'RESULTS STORED IN FILE= ',OUTFILE
155:     WRITE(*,*) 'OPEN BY MSWORD OR EDIT OR ANY OTHER EDITOR'
156:     WRITE(*,*) ' '
157:     WRITE(*,*) 'NOTE:VECTORS OF CORRELATIONS & INDEX(BOTH TOGETHER) ARE
158: & IDETERMINATE FOR SIGN & MAY BE MULTIPLIED BY (-1) IF NEEDED'
159:     WRITE(*,*) 'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
160: &THE INDEX(I) TOO IS TRANSFORMED TO -INDEX(I) FOR ALL I'
161:     WRITE(9,*) ' '
162:     WRITE(9,*) 'NOTE: VECTORS OF CORRELATIONS AND INDEX (BOTH TOGETHER)
163: & ARE IDETERMINATE FOR SIGN AND MAY BE MULTIPLIED BY (-1) IF NEEDED'
164:     WRITE(9,*) 'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
165: &THE INDEX(I) TOO IS TRANSFORMED TO -INDEX(I) FOR ALL I'
166:     CALL DORANK(FRANK,NOB)
167:     DO I=1,NOB
168:     ZDAT(I,1)=FRANK(I)
169:     ENDDO
170:     IF(NCOR.EQ.0) THEN
171:     CALL CORREL(ZDAT,NOB,MVAR+1,RMAT)
172:     ELSE
173:     CALL DOCORA(ZDAT,NOB,MVAR+1,RMAT)
174:     ENDIF
175:     WRITE(9,*) '----- CORRELATION MATRIX -----'
176:     WRITE(*,*) '----- CORRELATION MATRIX -----'
177:     DO I=1,MVAR+1
178:     WRITE(9,1)(RMAT(I,J),J=1,MVAR+1)
179:     WRITE(*,1)(RMAT(I,J),J=1,MVAR+1)
180:     ENDDO
181:     1 FORMAT(8F10.6)
182:     WRITE(9,*) '===== '
183:     WRITE(*,*) '===== '
184:     WRITE(9,*) 'VARIABLES: 1ST IS THE INDEX AND 2ND THE RANK OF INDEX'
185:     WRITE(*,*) 'VARIABLES: 1ST IS THE INDEX AND 2ND THE RANK OF INDEX'
186:     WRITE(9,*) '===== '
187:     WRITE(*,*) '===== '
188:     DO I=1,NOB
189:     IF(MRNK.EQ.1) THEN
190:     QIND(I)=0.D0
191:     DO J=1,MVAR
192:     QIND(I)=QIND(I)+ZDAT(I,J+1)*XX(NOPT,J)
193:     ENDDO
194:     ENDIF
195:     WRITE(9,2)I,QIND(I),(ZDAT(I,J),J=1,MVAR+1)
196:     WRITE(*,2)I,QIND(I),(ZDAT(I,J),J=1,MVAR+1)
197:     ENDDO
198:     2 FORMAT(I4,F12.6,9F7.2)
199:     SR2=0.D0
200:     IF(NORM.LE.2) THEN
201:     DO J=2,MVAR+1

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202:      SR2=SR2+DABS (RMAT (1, J) ) **NORM
203:      ENDDO
204:      IF (NORM.EQ.2) THEN
205:      WRITE (9, *) 'SUM OF SQUARE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
206:      WRITE (*, *) 'SUM OF SQUARE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
207:      ENDIF
208:      IF (NORM.EQ.1) THEN
209:      WRITE (9, *) 'SUM OF ABSOLUTE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
210:      WRITE (*, *) 'SUM OF ABSOLUTE OF CORRELATION R(RANK(INDEX),VAR)=' ,SR2
211:      ENDIF
212:      ELSE ! GIVES MAXIMAL CORRELATION
213:      SR2=1.D0
214:      DO J=2, MVAR+1
215:      IF (SR2.GT. DABS (RMAT (1, J) ) ) SR2=DABS (RMAT (1, J) )
216:      ENDDO
217:      WRITE (9, *) 'MAXIMIN CORRELATION R(RANK(INDEX),VAR)=' ,SR2
218:      WRITE (*, *) 'MAXIMIN CORRELATION R(RANK(INDEX),VAR)=' ,SR2
219:      ENDIF
220:      CLOSE (9)
221:      WRITE (*, *) 'THE JOB IS OVER'
222:      END
223: C
224:      SUBROUTINE RPS (M, ABEST, FBEST, G1)
225: C      PROGRAM TO FIND GLOBAL MINIMUM BY REPULSIVE PARTICLE SWARM METHOD
226: C      WRITTEN BY SK MISHRA, DEPT. OF ECONOMICS, NEHU, SHILLONG (INDIA)
227: C
228:      PARAMETER (N=100, NN=30, MX=100, NSTEP=7, ITRN=10000, NSIGMA=1, ITOP=1)
229:      PARAMETER (NPRN=50) ! DISPLAYS RESULTS AT EVERY 500 TH ITERATION
230: C      PARAMETER (N=50, NN=25, MX=100, NSTEP=9, ITRN=10000, NSIGMA=1, ITOP=3)
231: C      PARAMETER (N=100, NN=15, MX=100, NSTEP=9, ITRN=10000, NSIGMA=1, ITOP=3)
232: C      IN CERTAIN CASES THE ONE OR THE OTHER SPECIFICATION WORKS BETTER
233: C      DIFFERENT SPECIFICATIONS OF PARAMETERS MAY SUIT DIFFERENT TYPES
234: C      OF FUNCTIONS OR DIMENSIONS - ONE HAS TO DO SOME TRIAL AND ERROR
235: C
236: C      N = POPULATION SIZE. IN MOST OF THE CASES N=30 IS OK. ITS VALUE
237: C      MAY BE INCREASED TO 50 OR 100 TOO. THE PARAMETER NN IS THE SIZE OF
238: C      RANDOMLY CHOSEN NEIGHBOURS. 15 TO 25 (BUT SUFFICIENTLY LESS THAN
239: C      N) IS A GOOD CHOICE. MX IS THE MAXIMAL SIZE OF DECISION VARIABLES.
240: C      IN F(X1, X2, ..., XM) M SHOULD BE LESS THAN OR EQUAL TO MX. ITRN IS
241: C      THE NO. OF ITERATIONS. IT MAY DEPEND ON THE PROBLEM. 200 (AT LEAST)
242: C      TO 500 ITERATIONS MAY BE GOOD ENOUGH. BUT FOR FUNCTIONS LIKE
243: C      ROSENBROCKOR GRIEWANK OF LARGE SIZE (SAY M=30) IT IS NEEDED THAT
244: C      ITRN IS LARGE, SAY 5000 OR EVEN 10000.
245: C      SIGMA INTRODUCES PERTURBATION & HELPS THE SEARCH JUMP OUT OF LOCAL
246: C      OPTIMA. FOR EXAMPLE : RASTRIGIN FUNCTION OF DMENSION 30 OR LARGER
247: C      NSTEP DOES LOCAL SEARCH BY TUNNELLING AND WORKS WELL BETWEEN 5 AND
248: C      15, WHICH IS MUCH ON THE HIGHER SIDE.
249: C      ITOP <=1 (RING); ITOP=2 (RING AND RANDOM); ITOP=>3 (RANDOM)
250: C      NSIGMA=0 (NO CHAOTIC PERTURBATION); NSIGMA=1 (CHAOTIC PERTURBATION)
251: C      NOTE THAT NSIGMA=1 NEED NOT ALWAYS WORK BETTER (OR WORSE)
252: C      SUBROUTINE FUNC( ) DEFINES OR CALLS THE FUNCTION TO BE OPTIMIZED.
253:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
254:      COMMON /RNDM/ IU, IV
255:      COMMON /KFF/KF, NFCALL, FTIT
256:      INTEGER IU, IV
257:      CHARACTER *70 FTIT
258:      DIMENSION X (N, MX), V (N, MX), A (MX), VI (MX), TIT (50), ABEST (*)
259:      DIMENSION XX (N, MX), F (N), V1 (MX), V2 (MX), V3 (MX), V4 (MX), BST (MX)
260: C      A1 A2 AND A3 ARE CONSTANTS AND W IS THE INERTIA WEIGHT.
261: C      OCCASIONALLY, TINKERING WITH THESE VALUES, ESPECIALLY A3, MAY BE
262: C      NEEDED.
263:      DATA A1, A2, A3, W, SIGMA / .5D00, .5D00, .0005D00, .5D00, 1.D-03/
264:      EPSILON=1.D-12 ! ACCURACY NEEDED FOR TERMINATON
265: C      -----CHOOSING THE TEST FUNCTION -----
266:      CALL FSELECT (KF, M, FTIT)
267: C
268:      FFMIN=1.D30

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269:      LCOUNT=0
270:      NFCALL=0
271:      WRITE(*,*) '4-DIGITS SEED FOR RANDOM NUMBER GENERATION'
272:      READ(*,*) IU
273:      DATA FMIN /1.0E30/
274: C      GENERATE N-SIZE POPULATION OF M-TUPLE PARAMETERS X(I,J) RANDOMLY
275:      DO I=1,N
276:          DO J=1,M
277:              CALL RANDOM(RAND)
278:              X(I,J)=RAND
279: C      WE GENERATE RANDOM(-5,5). HERE MULTIPLIER IS 10. TINKERING IN SOME
280: C      CASES MAY BE NEEDED
281:          ENDDO
282:          F(I)=1.0D30
283:      ENDDO
284: C      INITIALISE VELOCITIES V(I) FOR EACH INDIVIDUAL IN THE POPULATION
285:      DO I=1,N
286:          DO J=1,M
287:              CALL RANDOM(RAND)
288:              V(I,J)=(RAND-0.5D+00)
289: C          V(I,J)=RAND
290:      ENDDO
291:      ENDDO
292:      DO 100 ITER=1, ITRN
293:      WRITE(*,*) 'ITERATION=', ITER
294: C      LET EACH INDIVIDUAL SEARCH FOR THE BEST IN ITS NEIGHBOURHOOD
295:          DO I=1,N
296:              DO J=1,M
297:                  A(J)=X(I,J)
298:                  VI(J)=V(I,J)
299:              ENDDO
300:              CALL LSRCH(A,M,VI,NSTEP,FI)
301:              IF(FI.LT.F(I)) THEN
302:                  F(I)=FI
303:                  DO IN=1,M
304:                      BST(IN)=A(IN)
305:                  ENDDO
306: C          F(I) CONTAINS THE LOCAL BEST VALUE OF FUNCTION FOR ITH INDIVIDUAL
307: C          XX(I,J) IS THE M-TUPLE VALUE OF X ASSOCIATED WITH LOCAL BEST F(I)
308:              DO J=1,M
309:                  XX(I,J)=A(J)
310:              ENDDO
311:          ENDDIF
312:      ENDDO
313: C      NOW LET EVERY INDIVIDUAL RANDOMLY COSULT NN(<<N) COLLEAGUES AND
314: C      FIND THE BEST AMONG THEM
315:      DO I=1,N
316: C      -----
317:      IF(ITOP.GE.3) THEN
318: C      RANDOM TOPOLOGY *****
319: C      CHOOSE NN COLLEAGUES RANDOMLY AND FIND THE BEST AMONG THEM
320:          BEST=1.0D30
321:          DO II=1,NN
322:              CALL RANDOM(RAND)
323:              NF=INT(RAND*N)+1
324:              IF(BEST.GT.F(NF)) THEN
325:                  BEST=F(NF)
326:                  NFBEST=NF
327:              ENDDIF
328:          ENDDO
329:      ENDDIF
330: C      -----
331:      IF(ITOP.EQ.2) THEN
332: C      RING + RANDOM TOPOLOGY *****
333: C      REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
334:          BEST=1.0D30
335:          CALL NEIGHBOR(I,N,I1,I3)

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336:         DO II=1,NN
337:             IF (II.EQ.1) NF=I1
338:             IF (II.EQ.2) NF=I
339:             IF (II.EQ.3) NF=I3
340:             IF (II.GT.3) THEN
341:                 CALL RANDOM(RAND)
342:                 NF=INT(RAND*N)+1
343:             ENDIF
344:             IF (BEST.GT.F(NF)) THEN
345:                 BEST=F(NF)
346:                 NFBEST=NF
347:             ENDIF
348:         ENDDO
349:     ENDIF
350: C-----
351:     IF (ITOP.LE.1) THEN
352: C     RING TOPOLOGY *****
353: C     REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
354:         BEST=1.0D30
355:         CALL NEIGHBOR(I,N,I1,I3)
356:         DO II=1,3
357:             IF (II.NE.I) THEN
358:                 IF (II.EQ.1) NF=I1
359:                 IF (II.EQ.3) NF=I3
360:                 IF (BEST.GT.F(NF)) THEN
361:                     BEST=F(NF)
362:                     NFBEST=NF
363:                 ENDIF
364:             ENDIF
365:         ENDDO
366:     ENDIF
367: C-----
368: C     IN THE LIGHT OF HIS OWN AND HIS BEST COLLEAGUES EXPERIENCE, THE
369: C     INDIVIDUAL I WILL MODIFY HIS MOVE AS PER THE FOLLOWING CRITERION
370: C     FIRST, ADJUSTMENT BASED ON ONES OWN EXPERIENCE
371: C     AND OWN BEST EXPERIENCE IN THE PAST (XX(I))
372:         DO J=1,M
373:             CALL RANDOM(RAND)
374:             V1(J)=A1*RAND*(XX(I,J)-X(I,J))
375:
376: C     THEN BASED ON THE OTHER COLLEAGUES BEST EXPERIENCE WITH WEIGHT W
377: C     HERE W IS CALLED AN INERTIA WEIGHT 0.01< W < 0.7
378: C     A2 IS THE CONSTANT NEAR BUT LESS THAN UNITY
379:             CALL RANDOM(RAND)
380:             V2(J)=V(I,J)
381:             IF (F(NFBEST).LT.F(I)) THEN
382:                 V2(J)=A2*W*RAND*(XX(NFBEST,J)-X(I,J))
383:             ENDIF
384: C     THEN SOME RANDOMNESS AND A CONSTANT A3 CLOSE TO BUT LESS THAN UNITY
385:             CALL RANDOM(RAND)
386:             RND1=RAND
387:             CALL RANDOM(RAND)
388:             V3(J)=A3*RAND*W*RND1
389: C     V3(J)=A3*RAND*W
390: C     THEN ON PAST VELOCITY WITH INERTIA WEIGHT W
391:             V4(J)=W*V(I,J)
392: C     FINALLY A SUM OF THEM
393:             V(I,J)= V1(J)+V2(J)+V3(J)+V4(J)
394:         ENDDO
395:     ENDDO
396: C     CHANGE X
397:     DO I=1,N
398:     DO J=1,M
399:     RANDB=0.D00
400: C-----
401:     IF (NSIGMA.EQ.1) THEN
402:         CALL RANDOM(RAND) ! FOR CHAOTIC PERTURBATION

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403:      IF (DABS (RAND-.5D00) .LT. SIGMA) RANDS=RAND-0.5D00
404: C      SIGMA CONDITIONED RANDS INTRODUCES CHAOTIC ELEMENT IN TO LOCATION
405: C      IN SOME CASES THIS PERTURBATION HAS WORKED VERY EFFECTIVELY WITH
406: C      PARAMETER (N=100, NN=15, MX=100, NSTEP=9, ITRN=100000, NSIGMA=1, ITOP=2)
407:      ENDIF
408: C      -----
409:      X(I, J)=X(I, J)+V(I, J)*(1.D00+RANDS)
410:      ENDDO
411:      ENDDO
412:      DO I=1, N
413:          IF (F(I) .LT. FMIN) THEN
414:              FMIN=F(I)
415:              II=I
416:              DO J=1, M
417:                  BST(J)=XX(II, J)
418:              ENDDO
419:          ENDIF
420:      ENDDO
421:
422:      IF (LCOUNT.EQ. NPRN) THEN
423:          LCOUNT=0
424:          WRITE (*, *) 'OPTIMAL SOLUTION UPTO THIS (FUNCTION CALLS=', NFCALL, ')'
425:          WRITE (*, *) 'X = ', (BST(J), J=1, M), ' MIN F = ', FMIN
426: C      WRITE (*, *) 'NO. OF FUNCTION CALLS = ', NFCALL
427:          DO J=1, M
428:              ABEST(J)=BST(J)
429:          ENDDO
430:          IF (DABS (FFMIN-FMIN) .LT. EPSILON) GOTO 999
431:          FFMIN=FMIN
432:      ENDIF
433:      LCOUNT=LCOUNT+1
434: 100 CONTINUE
435: 999 WRITE (*, *) '-----'
436:      DO I=1, N
437:          IF (F(I) .LT. FBEST) THEN
438:              FBEST=F(I)
439:              DO J=1, M
440:                  ABEST(J)=XX(I, J)
441:              ENDDO
442:          ENDIF
443:      ENDDO
444:      CALL FUNC (ABEST, M, FBEST)
445:      CALL GINI (F, N, G1)
446:      WRITE (*, *) 'FINAL X = ', (BST(J), J=1, M), ' FINAL MIN F = ', FMIN
447:      WRITE (*, *) 'COMPUTATION OVER:FOR ', FTIT
448:      WRITE (*, *) 'NO. OF VARIABLES=', M, ' END.'
449:      RETURN
450:      END
451: C      -----
452:      SUBROUTINE LSRCH (A, M, VI, NSTEP, FI)
453:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
454:      CHARACTER *70 FTIT
455:      COMMON /KFF/KF, NFCALL, FTIT
456:      COMMON /RNDM/IU, IV
457:      INTEGER IU, IV
458:      DIMENSION A (*), B (100), VI (*)
459:      AMN=1.0D30
460:      DO J=1, NSTEP
461:          DO JJ=1, M
462:              B (JJ)=A (JJ) + (J- (NSTEP/2)-1) *VI (JJ)
463:          ENDDO
464:          CALL FUNC (B, M, FI)
465:          IF (FI .LT. AMN) THEN
466:              AMN=FI
467:              DO JJ=1, M
468:                  A (JJ)=B (JJ)
469:              ENDDO

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470:      ENDIF
471:      ENDDO
472:      FI=AMN
473:      RETURN
474:      END
475: C -----
476: C THIS SUBROUTINE IS NEEDED IF THE NEIGHBOURHOOD HAS RING TOPOLOGY
477: C EITHER PURE OR HYBRIDIZED
478:      SUBROUTINE NEIGHBOR (I, N, J, K)
479:      IF (I-1.GE.1 .AND. I.LT.N) THEN
480:      J=I-1
481:      K=I+1
482:      ELSE
483:      IF (I-1.LT.1) THEN
484:      J=N-I+1
485:      K=I+1
486:      ENDIF
487:      IF (I.EQ.N) THEN
488:      J=I-1
489:      K=1
490:      ENDIF
491:      ENDIF
492:      RETURN
493:      END
494: C -----
495: C RANDOM NUMBER GENERATOR (UNIFORM BETWEEN 0 AND 1 - BOTH EXCLUSIVE)
496:      SUBROUTINE RANDOM(RAND1)
497:      DOUBLE PRECISION RAND1
498:      COMMON /RNDM/IU, IV
499:      INTEGER IU, IV
500:      IV=IU*65539
501:      IF (IV.LT.0) THEN
502:      IV=IV+2147483647+1
503:      ENDIF
504:      RAND=IV
505:      IU=IV
506:      RAND=RAND*0.4656613E-09
507:      RAND1= DBLE(RAND)
508:      RETURN
509:      END
510: C -----
511:      SUBROUTINE GINI (F, N, G)
512:      PARAMETER (K=1) !K=1 GINI COEFFICIENT; K=2 COEFFICIENT OF VARIATION
513: C THIS PROGRAM COMPUTES MEASURE OF INEQUALITY
514: C IF K =1 GET THE GINI COEFFICIENT. IF K=2 GET COEFF OF VARIATIONE
515:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
516:      DIMENSION F(*)
517:      S=0.D0
518:      DO I=1, N
519:      S=S+F (I)
520:      ENDDO
521:      S=S/N
522:      H=0.D00
523:      DO I=1, N-1
524:      DO J=I+1, N
525:      H=H+ (DABS (F (I) -F (J) ) ) **K
526:      ENDDO
527:      ENDDO
528:      H=(H/ (N**2) ) ** (1.D0/K) ! FOR K=1 H IS MEAN DEVIATION;
529: C FOR K=2 H IS STANDARD DEVIATION
530:      WRITE (*, *) 'MEASURES OF DISPERSION AND CENTRAL TENDENCY = ', G, S
531:      G=DEXP (-H) ! G IS THE MEASURE OF EQUALITY (NOT GINI OR CV)
532: C G=H/DABS(S) !IF S NOT ZERO, K=1 THEN G=GINI, K=2 G=COEFF VARIATION
533:      RETURN
534:      END
535: C -----
536:      SUBROUTINE FSELECT (KF, M, FTIT)

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537: C      THE PROGRAM REQUIRES INPUTS FROM THE USER ON THE FOLLOWING -----
538: C      (1) FUNCTION CODE (KF), (2) NO. OF VARIABLES IN THE FUNCTION (M);
539: CHARACTER *70 TIT(100), FTIT
540: NFN=1
541: KF=1
542: WRITE (*, *) '-----'
543: DATA TIT(1) / 'CONSTRUCTION OF INDEX FROM M VARIABLES ' /
544: C      -----
545: DO I=1, NFN
546: WRITE (*, *) TIT(I)
547: ENDDO
548: WRITE (*, *) '-----'
549: WRITE (*, *) 'SPECIFY NO. OF VARIABLES [MVAR] HERE ALSO ?'
550: READ (*, *) M
551: FTIT=TIT(KF) ! STORE THE NAME OF THE CHOSEN FUNCTION IN FTIT
552: RETURN
553: END
554: C      -----
555: SUBROUTINE FUNC(X, M, F)
556: C      TEST FUNCTIONS FOR GLOBAL OPTIMIZATION PROGRAM
557: IMPLICIT DOUBLE PRECISION (A-H, O-Z)
558: COMMON /RNDM/IU, IV
559: COMMON /KFF/KF, NFCALL, FTIT
560: INTEGER IU, IV
561: DIMENSION X(*)
562: CHARACTER *70 FTIT
563: NFCALL=NFCALL+1 ! INCREMENT TO NUMBER OF FUNCTION CALLS
564: C      KF IS THE CODE OF THE TEST FUNCTION
565: IF (KF.EQ.1) THEN
566: CALL CORD(M, X, F)
567: RETURN
568: ENDIF
569: C      =====
570: WRITE (*, *) 'FUNCTION NOT DEFINED. PROGRAM ABORTED'
571: STOP
572: END
573: C      -----
574: SUBROUTINE CORD(M, X, F)
575: PARAMETER (NOB=30, MVAR=5) ! CHANGE THE PARAMETERS HERE AS NEEDED.
576: C      -----
577: C      NOB=NO. OF OBSERVATIONS (CASES) & MVAR= NO. OF VARIABLES
578: IMPLICIT DOUBLE PRECISION (A-H, O-Z)
579: COMMON /RNDM/IU, IV
580: COMMON /CORDAT/CDAT (NOB, MVAR), QIND (NOB), R (MVAR), ENTROPY, NORM, NCOR
581: COMMON /GETRANK/MRNK
582: INTEGER IU, IV
583: DIMENSION X(*), Z (NOB, 2)
584: DO I=1, M
585: IF (X(I).LT.-1.0D0.OR.X(I).GT.1.0D0) THEN
586: CALL RANDOM(RAND)
587: X(I)=(RAND-0.5D0)*2
588: ENDIF
589: ENDDO
590: XNORM=0.D0
591: DO J=1, M
592: XNORM=XNORM+X(J)**2
593: ENDDO
594: XNORM=DSQRT(XNORM)
595: DO J=1, M
596: X(J)=X(J)/XNORM
597: ENDDO
598: C      CONSTRUCT INDEX
599: DO I=1, NOB
600: QIND(I)=0.D0
601: DO J=1, M
602: QIND(I)=QIND(I)+CDAT(I, J)*X(J)
603: ENDDO

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604:      ENDDO
605: C -----
606:      !FIND THE RANK OF QIND
607:      IF (MRNK.EQ.1) CALL DORANK(QIND,NOB)
608: C -----
609: C      COMPUTE CORRELATIONS
610:      DO I=1,NOB
611:      Z(I,1)=QIND(I)
612:      ENDDO
613:      DO J=1,M
614:      DO I=1,NOB
615:      Z(I,2)=CDAT(I,J)
616:      ENDDO
617:      IF (NCOR.EQ.0) THEN
618:      CALL CORLN(Z,NOB,RHO)
619:      ELSE
620:      CALL CORA(Z,NOB,RHO)
621:      ENDF
622:      R(J)=RHO
623:      ENDDO
624:      IF (ENTROPY.EQ.0.D0) THEN
625: C ----- MAXIMIN SOLUTION -----
626:      IF (NORM.GT.2) THEN
627:      FR=DABS(R(1))
628:      DO J=2,M
629:      IF (FR.GT.DABS(R(J))) FR= DABS(R(J))
630:      ENDDO
631:      F=FR
632:      ENDF
633: C ----- FOR NORM =1 OR 2 -----
634:      IF (NORM.LE.2) THEN
635:      F=0.D0
636:      DO J=1,M
637:      F=F+DABS(R(J))**NORM
638:      ENDDO
639:      ENDF
640: C -----
641:      ELSE
642:      IF (ENTROPY.NE.0.D0) THEN
643: C ----- ENTROPY MAXIMIZATION -----
644:      ENT=0.D0
645:      DO J=1,M
646:      ENT=ENT+DABS(R(J))
647:      ENDDO
648:      F=ENT*DLOG(ENT)
649:      DO J=1,M
650:      FX=DABS(R(J))
651:      F=F+ (FX/ENT)*DLOG(FX/ENT)
652:      ENDDO
653:      ENDF
654:      ENDF
655: C -----
656:      F=-F
657:      RETURN
658:      END
659:      SUBROUTINE CORLN(Z,NOB,RHO)
660: C ----- NOB = NO. OF CASES -----
661:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
662:      DIMENSION Z(NOB,2),AV(2),SD(2)
663:      DO J=1,2
664:      AV(J)=0.D0
665:      SD(J)=0.D0
666:      DO I=1,NOB
667:      AV(J)=AV(J)+Z(I,J)
668:      SD(J)=SD(J)+Z(I,J)**2
669:      ENDDO
670:      ENDDO

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671:      DO J=1,2
672:      AV(J)=AV(J)/NOB
673:      SD(J)=DSQRT(SD(J)/NOB-AV(J)**2)
674:      ENDDO
675: C      WRITE(*,*) 'AV AND SD ', AV(1),AV(2),SD(1),SD(2)
676:      RHO=0.D0
677:      DO I=1,NOB
678:      RHO=RHO+(Z(I,1)-AV(1))*(Z(I,2)-AV(2))
679:      ENDDO
680:      RHO=(RHO/NOB)/(SD(1)*SD(2))
681:      RETURN
682:      END
683: C -----
684:      SUBROUTINE CORA(Z,N,R)
685: C      COMPUTING BRADLEY'S ABSOLUTE CORRELATION MATRIX
686: C      BRADLEY, C. (1985) "THE ABSOLUTE CORRELATION", THE MATHEMATICAL
687: C      GAZETTE, 69(447): 12-17.
688:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
689:      DIMENSION Z(N,2),X(N),Y(N)
690: C -----
691: C      PUT Z INTO X AND Y
692:      DO I=1,N
693:      X(I)=Z(I,1)
694:      Y(I)=Z(I,2)
695:      ENDDO
696: C      ARRANGE X ANY IN AN ASCENDING ORDER
697:      DO I=1,N-1
698:      DO II=I+1,N
699:      IF(X(I).GT.X(II)) THEN
700:      TEMP=X(I)
701:      X(I)=X(II)
702:      X(II)=TEMP
703:      ENDIF
704:      IF(Y(I).GT.Y(II)) THEN
705:      TEMP=Y(I)
706:      Y(I)=Y(II)
707:      Y(II)=TEMP
708:      ENDIF
709:      ENDDO
710:      ENDDO
711: C      FIND MEDIAN
712:      IF(INT(N/2).EQ.N/2.D0) THEN
713:      XMED=(X(N/2)+X(N/2+1))/2.D0
714:      YMED=(Y(N/2)+Y(N/2+1))/2.D0
715:      ENDIF
716:      IF(INT(N/2).NE.N/2.D0) THEN
717:      XMED=X(N/2+1)
718:      YMED=Y(N/2+1)
719:      ENDIF
720: C      SUBTRACT RESPECTIVE MEDIANS FROM X AND Y AND FIND ABS DEVIATIONS
721:      VX=0.D0
722:      VY=0.D0
723:      DO I=1,N
724:      X(I)=X(I)-XMED
725:      Y(I)=Y(I)-YMED
726:      VX=VX+DABS(X(I))
727:      VY=VY+DABS(Y(I))
728:      ENDDO
729: C      SCALE THE VARIABLES X AND Y SUCH THAT VX=VY
730:      IF(VX.EQ.0.D0.OR.VY.EQ.0.D0) THEN
731:      R=0.D0
732:      RETURN
733:      ENDIF
734:      DO I=1,N
735:      X(I)=X(I)*VY/VX
736:      ENDDO
737: C      COMPUTE CORRELATION COEFFICIENT

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738:      VZ=0.D0
739:      R=0.D0
740:      DO I=1,N
741:      VZ=VZ+DABS(X(I))+DABS(Y(I))
742:      R=R+DABS(X(I)+Y(I))-DABS(X(I)-Y(I))
743:      ENDDO
744:      R=R/VZ
745:      RETURN
746:      END
747: C -----
748:      SUBROUTINE DORANK(X,N) ! N IS THE NUMBER OF OBSERVATIONS
749:      PARAMETER (NRL=0) ! THIS VALUE IS TO BE SET BY THE USER
750: C      !THE VALUE OF NRL DECIDES THE SCHEME OF RANKINGS
751: C      !THIS PROGRAM RETURNS RANK-ORDER OF A GIVEN VECTOR
752:      PARAMETER (MXD=1000) ! MXD IS MAX DIMENSION FOR TEMPORARY VARIABLES
753:      ! THAT ARE LOCAL AND DO NOT GO TO THE INVOKING PROGRAM
754:      ! X IS THE VARIABLE TO BE SUBSTITUTED BY ITS RANK VALUES
755: C      NRULE=0 FOR ORDINAL RANKING (1-2-3-4 RULE);
756: C      NRULE=1 FOR DENSE RANKING (1-2-2-3 RULE);
757: C      NRULE=2 FOR STANDARD COMPETITION RANKING (1-2-2-4 RULE);
758: C      NRULE=3 FOR MODIFIED COMPETITION RANKING (1-3-3-4 RULE);
759: C      NRULE=4 FOR FRACTIONAL RANKING (1-2.5-2.5-4 RULE);
760:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
761:      DIMENSION X(N),NF(MXD),NCF(MXD),RANK(MXD),ID(MXD),XX(MXD)
762: C      GENERATE ID(I),I=1,N
763:      DO I=1,N
764:      ID(I)=I
765:      NF(I)=0
766:      ENDDO
767: C      ARRANGE DATA (X) AND THE IDS IN ASCENDING ORDER
768:      DO I=1,N-1
769:      DO II=I,N
770:      IF(X(II).LT.X(I)) THEN
771:      TEMP=X(I)
772:      X(I)=X(II)
773:      X(II)=TEMP
774:      ITEMP=ID(I)
775:      ID(I)=ID(II)
776:      ID(II)=ITEMP
777:      ENDIF
778:      ENDDO
779:      ENDDO
780: C      MAKE DISCRETE UNGROUPED FREQUENCY TABLE
781:      K=0
782:      J=1
783:      1 K=K+1
784:      XX(K)=X(J)
785:      NF(K)=0
786:      DO I=J,N
787:      IF(XX(K).EQ.X(I)) THEN
788:      NF(K)=NF(K)+1
789:      ELSE
790:      J=I
791:      IF(J.LE.N) THEN
792:      GOTO 1
793:      ELSE
794:      GOTO 2
795:      ENDIF
796:      ENDIF
797:      ENDDO
798:      2 KK=K
799:      DO K=1,KK
800:      IF(K.EQ.1) THEN
801:      NCF(K)=NF(K)
802:      ELSE
803:      NCF(K)=NCF(K-1)+NF(K)
804:      ENDIF

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805:      ENDDO
806:      DO I=1,N
807:      RANK(I)=1.D0
808:      ENDDO
809:
810:      IF(NRL.GT.4) THEN
811:      WRITE(*,*) 'RANKING RULE CODE GREATER THAN 4 NOT PERMITTED',NRL
812:      STOP
813:      ENDIF
814:
815:      IF(NRL.EQ.0) THEN
816:      DO I=1,N
817:      RANK(I)=I
818:      ENDDO
819:      ENDIF
820: C -----
821:      IF(NRL.GT.0) THEN
822:      DO K=1, KK
823:      IF(K.EQ.1) THEN
824:      K1=1
825:      ELSE
826:      K1=NCF(K-1)+1
827:      ENDIF
828:      K2=NCF(K)
829:      DO I=K1, K2
830:      SUM=0.D0
831:      DO II=K1, K2
832:      SUM=SUM+II
833:      ENDDO
834:      KX=(K2-K1+1)
835:      IF(NRL.EQ.1) RANK(I)=K ! DENSE RANKING (1-2-2-3 RULE)
836:      IF(NRL.EQ.2) RANK(I)=K1 ! STANDARD COMPETITION RANKING (1-2-2-4 RULE)
837:      IF(NRL.EQ.3) RANK(I)=K2 ! MODIFIED COMPETITION RANKING (1-3-3-4 RULE)
838:      IF(NRL.EQ.4) RANK(I)=SUM/KX ! FRACTIONAL RANKING (1-2.5-2.5-4 RULE)
839:      ENDDO
840:      ENDDO
841:      ENDIF
842: C -----
843:      DO I=1,N
844:      X(ID(I))=RANK(I) ! BRINGS THE DATA TO ORIGINAL SEQUENCE
845:      ENDDO
846:      RETURN
847:      END
848: C -----
849:      SUBROUTINE CORREL(X,N,M, RMAT)
850:      PARAMETER (NMX=30) !DO NOT CHANGE UNLESS NO. OF VARIABLES EXCEED 30
851:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
852:      DIMENSION X(N,M), RMAT(M,M), AV(NMX), SD(NMX)
853:      DO J=1,M
854:      AV(J)=0.D0
855:      SD(J)=0.D0
856:      DO I=1,N
857:      AV(J)=AV(J)+X(I,J)
858:      SD(J)=SD(J)+X(I,J)**2
859:      ENDDO
860:      AV(J)=AV(J)/N
861:      SD(J)=DSQRT(SD(J)/N-AV(J)**2)
862:      ENDDO
863:      DO J=1,M
864:      DO JJ=1,M
865:      RMAT(J,JJ)=0.D0
866:      DO I=1,N
867:      RMAT(J,JJ)=RMAT(J,JJ)+X(I,J)*X(I,JJ)
868:      ENDDO
869:      ENDDO
870:      ENDDO
871:      DO J=1,M

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872:      DO JJ=1,M
873:      RMAT(J,JJ)=RMAT(J,JJ)/N-AV(J)*AV(JJ)
874:      RMAT(J,JJ)=RMAT(J,JJ)/(SD(J)*SD(JJ))
875:      ENDDO
876:      ENDDO
877:      RETURN
878:      END
879: C -----
880:      SUBROUTINE DOCORA(ZDAT,N,M,RMAT)
881:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
882:      DIMENSION ZDAT(N,M),RMAT(M,M),Z(N,2)
883:      DO J=1,M-1
884:      DO JJ=J+1,M
885:      DO I=1,N
886:      Z(I,1)=ZDAT(I,J)
887:      Z(I,2)=ZDAT(I,JJ)
888:      ENDDO
889:      CALL CORA(Z,N,R)
890:      RMAT(J,JJ)=R
891:      RMAT(JJ,J)=R
892:      ENDDO
893:      ENDDO
894:      DO J=1,M
895:      RMAT(J,J)=1.D0
896:      ENDDO
897:      RETURN
898:      END
```